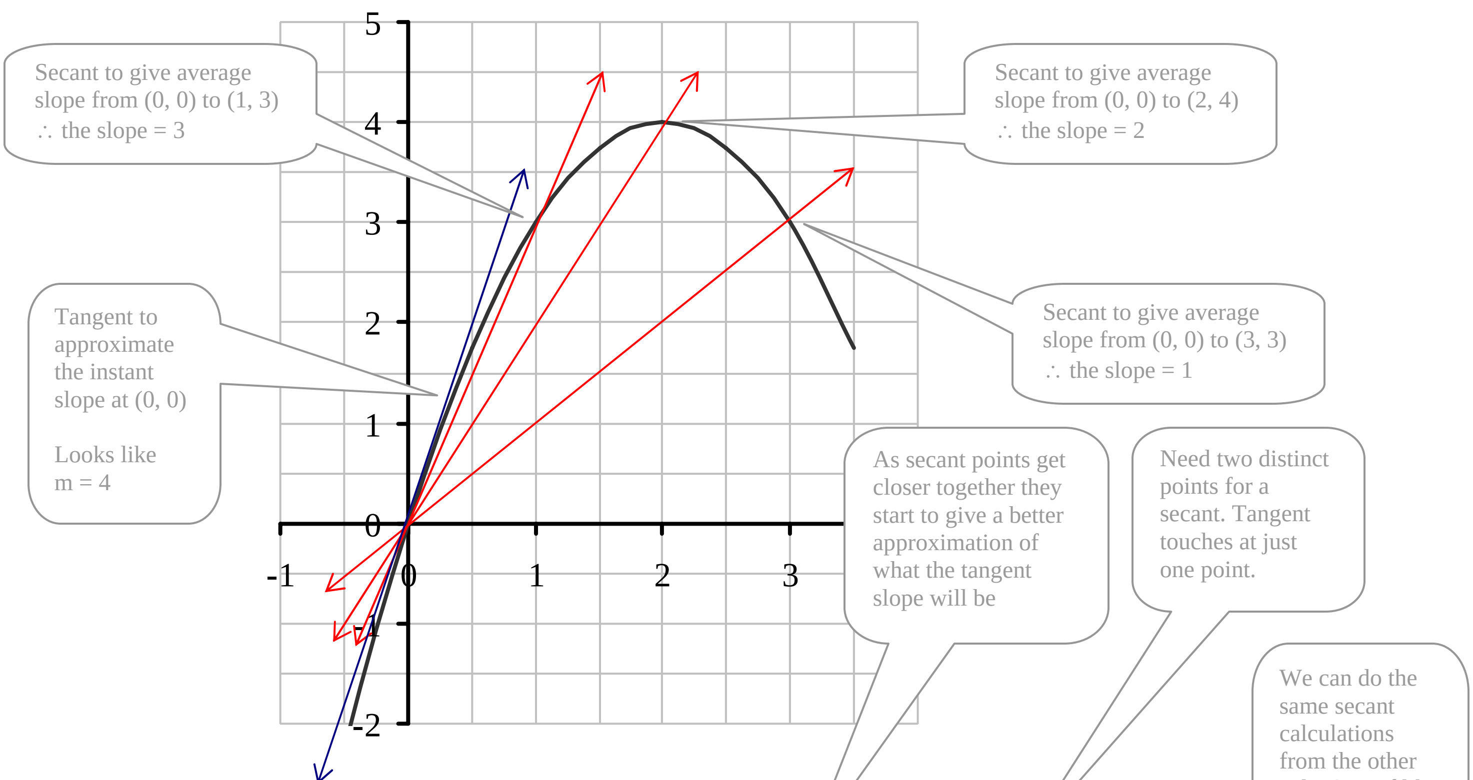
6.5A – Average versus Instantaneous Rate of Change

Secant is a line that joins two points on a curve. As such, it determines the average slope between these two points of the curve.

Tangent is a line that touches a curve at only one point. As such, it gives the instant slope of the curve at this one point. One can approximate the slope of the tangent by using a secant that has two points very close together on either side of the tangent point that they are interested in.

Ex. Consider the function: $f(x) = -(x - 2)^2 + 4$

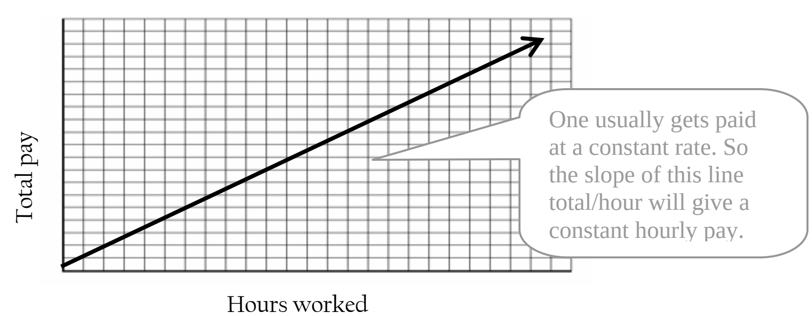


The following chart outlines the above observations

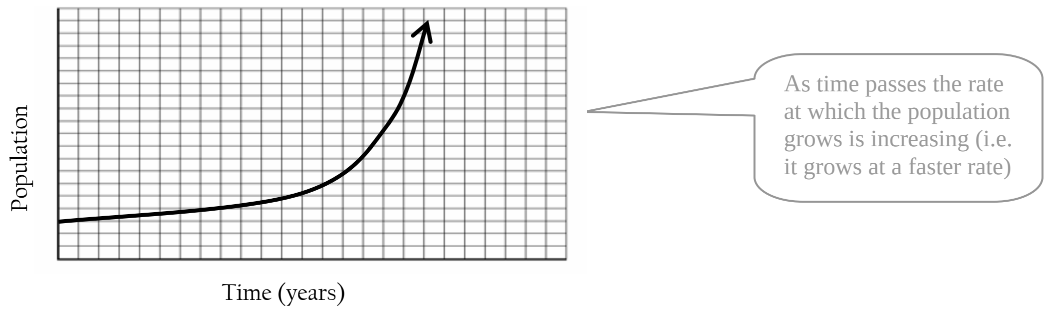
Point **x** on the function	3	2	1	0.5	0.25	0	-0.25	-0.5	-1
Average slope from origin (0,0) to **x**	1	2	3	3.5	3.8		4.2	4.5	5

The ***rate of change*** compares how one variable changes with respect to another. As the variables have different units we call this comparison a rate rather then a ratio. Graphically this rate can be defined as slope. Below are some common examples of how the rate of change can vary over the domain of a function.

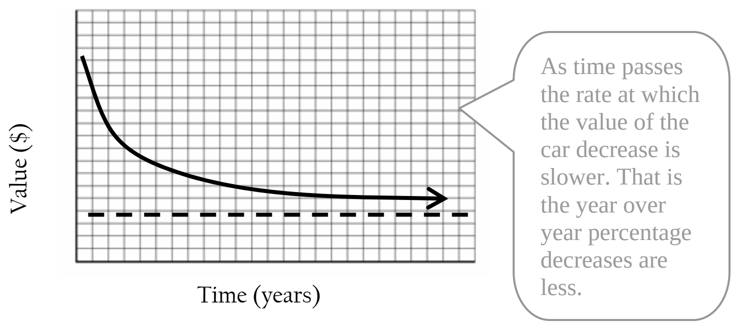
Ex. a) constant rate of change: $f(x) = mx + b$

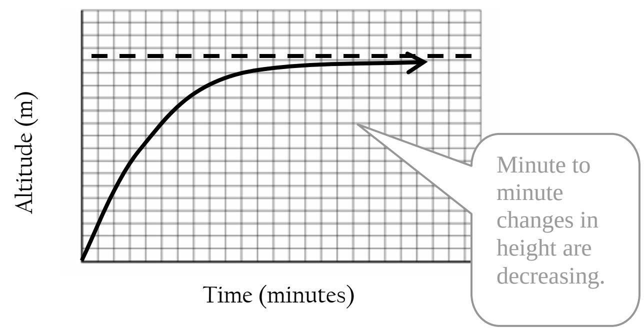


b) increasing rate of change (exponential, rational, ?)

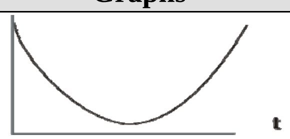
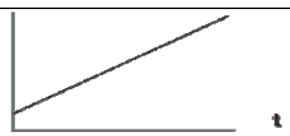
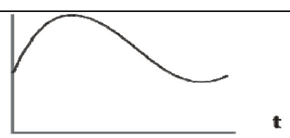
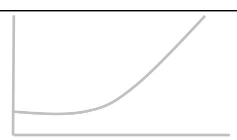
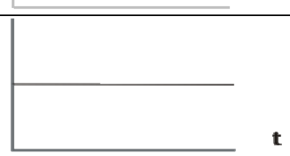


c) decreasing rate of change. (logarithmic, rational, ?)

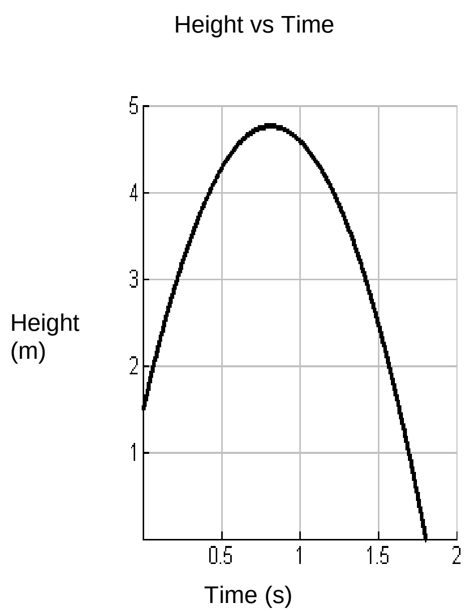




Example 1: Match the graph with the corresponding description and indicate (by circling your response) if the rate of change is zero, constant or changing.

Graphs		Description	Graph Match
A		1. A grade 12 student's height over the next 12 months.	____ zero constant changing
B		2. Money deposited on your 12th birthday grew slowly at first, then more quickly.	____ zero constant changing
C		3. Andrea walks quickly, slows to a stop, and then speeds up until she is travelling at the same speed as when she started.	____ Zero Constant changing
D		4. Over a one-month period the rate of growth for a sunflower is constant.	____ Zero Constant changing
E		5. Clara walks quickly and then slows to a stop. She then walks quickly and slows to a second stop. Clara then walks at a pace that is a little slower than when she started.	____ Zero Constant changing

Example 2: A height of a shot put can be modelled by the function $H(t) = -4.9t^2 + 8t + 1.5$, where h is the height in metres and t is the time in seconds. Graph shown below.



a) At what point do you think the shot put was travelling the fastest? (i.e. rate of change is greatest)

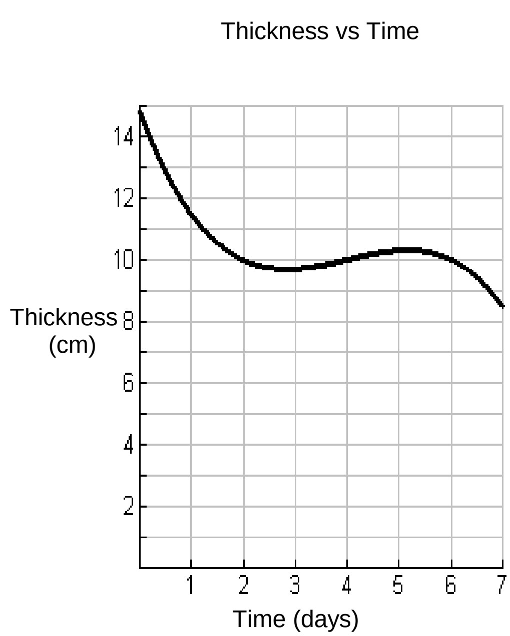
b) What factors did you use to make your inference?

c) Determine the average rate of change on a short interval near the point you chose in a).

d) Estimate the instantaneous rate of change at the point you chose in a).

e) Were your answers to the average rate of change the same as the instantaneous rate of change, if not why not?

Example 3: The thickness of the ice on a lake for one week is modelled by the function: $T(d) = -0.1d^3 + 1.2d^2 – 4.4d + 14.8$, where T is the thickness in cm and d is the number of days after December 31st. The graph of the function is shown.



a) When do you think the warmest day occurred during the week? Justify your answer.

b) Determine the average rate of change on a short interval near the point you chose in a).

c) Determine the instantaneous rate of change at the point you chose in a).

d) Were your answers to the average rate of change the same as the instantaneous rate of change, if not why not?

6.5A – Average versus Instantaneous Rate of Change Practice Questions

1. Determine if the below statement represent; average, close to instant or instant rates.

   a) Some road tolls in the U.S. give speeding tickets based on the time it takes you to travel between exits.
   b) A police officer pulls you over for speeding since her radar gun displays 130 km/hr.
   c) Canada’s population grew at a rate of 0.869% from 2006 to 2007.
   d) Roy Halliday’s fast ball was measured to have a velocity of 152 km/h.
   e) Your parents kept a growth chart from the time you were 1 until you were 5 years old. They have calculated that your growth rate in that period was 9 cm/year.
   f) In 1996, Hurricane Bertha had wind gusts up to185 km/h. At some times during Hurricane Bertha the wind was gusting at 100 km/h.
   g) Water is being poured into a container. The rate in which the water level increases between 0 and 5 seconds of the pour is 7 mm/sec.
   h) A CO_2 probe measures the rate of increase of atmospheric CO_2. The probe reads an increase of 1.7×10^{-8} ppm/sec.

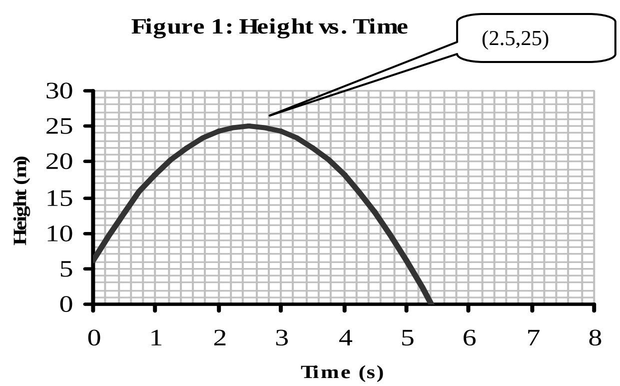
2. In general does the speedometer of a car measure average or instant rate of change. Describe a scenario in which the average speed and instant speed would be the same.

3. Sketch the curve $y = (½(x – 3))^2 + 4$

   a) Calculate the average slope (secant) from 0 to 3 seconds
   b) Draw a tangent at x = 7. Calculate the slope at this point
   c) Describe what happens to the rate of change from 0 to 3 seconds
   d) Describe what happens to the rate of change from 0 to 6 seconds
   e) Describe when rate of change is zero

4. The diagram below describes the height of a ball thrown into the air according to the formula $h(t) = -3(t-2.5)^2 + 25$. Determine;

   a) the maximum height ball reaches
   b) the initial height from which ball was thrown
   c) describe interval when ball’s height decreases
   d) the ***average rate*** at which the ball’s height changes from; i) 0 to 1s
      ii) 0 to 4s
   e) the ***instantaneous rate*** at which the ball’s height changes at; i) 4s
      ii) 2.5s



5. Sketch the function: $f(\theta) = 3\sin\left(\theta - \frac{\pi}{6}\right)$ on an interval $\left[\frac{\pi}{6}, \frac{7\pi}{6}\right]$

   a) Is the function increasing or decreasing on the interval $\pi/3$ to $2\pi/3$
   b) Draw the line through the points $f(\pi/3)$and $f(2\pi/3)$ and then find average slope.
   c) Describe how to find the instantaneous rate of change at $\pi/3$ What does this mean?

Answers 1.a) A **b)** I **c)** A **d)** C **e)** A **f)** I **g)** A **h)** I **2.** If drive at a constant speed **3.a)** 1.5 **b)** 4 **c)** decreases **d)** decrease till x-3 then increases when x>3 **e)** x=3 **4.a**) 25m **b)** 6m **c)** 2.5<t<5.4 **d) i)** 12m/s **ii)** 3m/s **e) i)** -7m/s **ii)** 0m/s **5.a)** increasing **b)** 4.5/π rad/θ **c)** about 2.56 rad/θ