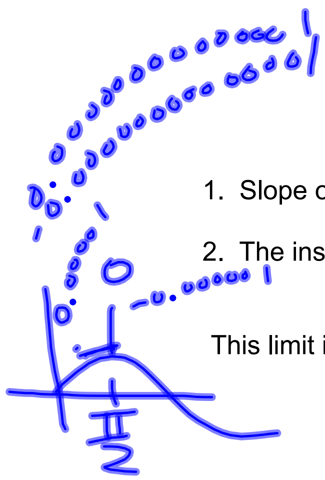
2.1 - The Derivative Function

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$



Does $f'(x)$ exist at $x=0$?

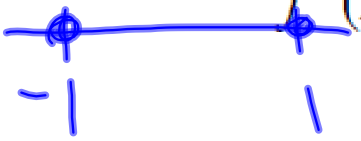
$f(x)=|x|$

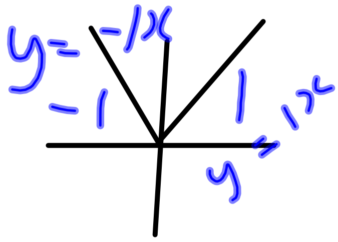
1. Slope of the tangent to the graph y=f(x) at the point x
2. The instantaneous rate of change at x.

This limit is super important to Calculus, so we give it a name!

THE DERIVATIVE!

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$





Example 1. Determine the derivative of f(x) = x^2 at x= 4

$f'(x) = 2x^1$

$f'(4) = 2 \cdot 4$

$= 8$

$f'(4) = \lim_{h \to 0} \frac{f(4+h) - f(4)}{h}$

$= \lim_{h \to 0} \frac{(4+h)^2 - (4)^2}{h}$

$= \lim_{h \to 0} \frac{\cancel{16} + 8h + h^2 - \cancel{16}}{h}$

$= \lim_{h \to 0} \frac{\cancel{h}(8+h)}{\cancel{h}}$

$= 8$

Example 2. Determine the derivative of f(x) = x^2 at an arbitrary value of x

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{x^2}}{h}$$

$f(x) = x^2$

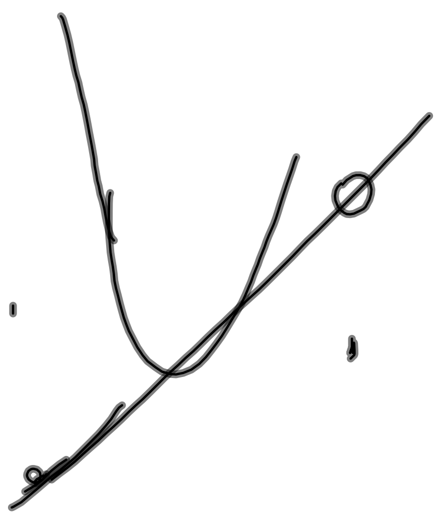
$f'(x) = 2x$

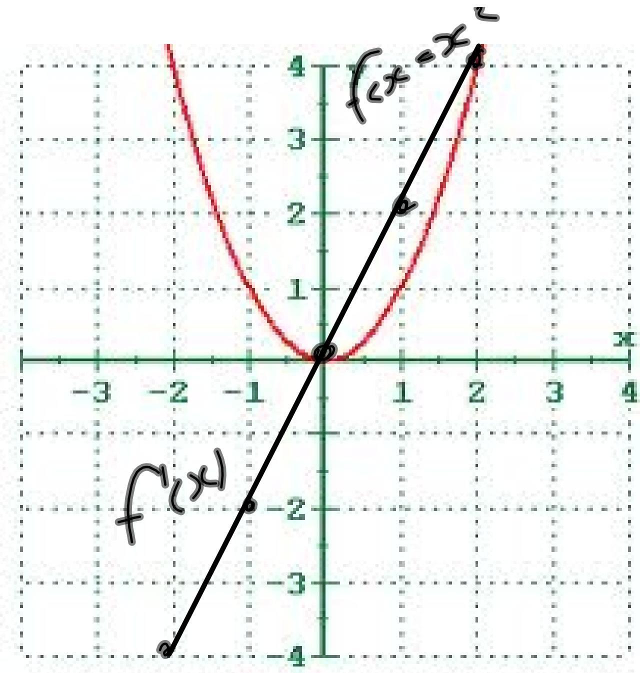
$$= \lim_{h \to 0} \frac{2xh + h^2}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(2x+h)}{\cancel{h}}$$

$$= \lim_{h \to 0} 2x + h$$

$$f'(x) = 2x$$




f(x)=x²
f'(x)
x
-3
-2
-1
1
2
3
4
4
3
2
1
-2
-3
-4

The derivative of f(x) at x=a is a number f'(a)

The derivative of f(x) for any arbitrary value x, is a function f'(x).

$f'(4) = 8$

$f'(x) = 2x$

$f(x) = 5x^4 + 3x^2 - 7$

$f'(x) = 20x^3 + 6x$

Example 3: Determine the derivative f'(t) of the function f(t) = $\sqrt{(t+3)}$

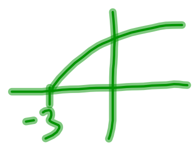
$$f'(t) = \lim_{h\to 0} \frac{\sqrt{t+h+3} - \sqrt{t+3}}{h}$$

$$f'(t) = \frac{1}{2\sqrt{t+3}}$$

$$= \lim_{h\to 0} \frac{\sqrt{t+h+3} - \sqrt{t+3}}{h} \cdot \frac{(\sqrt{} + \sqrt{})}{(\sqrt{} + \sqrt{})}$$

$$= \lim_{h\to 0} \frac{(\cancel{t}+h+\cancel{3}) - (\cancel{t}+\cancel{3})}{h(\sqrt{} + \sqrt{})}$$

$$= \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{} + \sqrt{})}$$



$$= \lim_{h\to 0} \frac{1}{\sqrt{t+h+3} + \sqrt{t+3}}$$

$$= \frac{1}{\sqrt{t+3} + \sqrt{t+3}}$$

$$= \frac{1}{2\sqrt{t+3}}$$

$t+3 > 0$

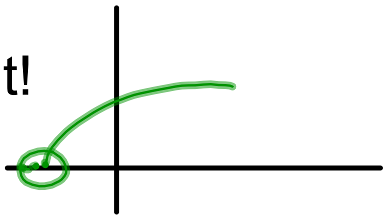
$t > -3$

Example 3b.

State the domain of f(x) $D: \{x \geq -3 \mid x \in \mathbb{R}\}$

State the domain of f'(x). $D: \{x > -3 \mid x \in \mathbb{R}\}$

Explain why they are different!



Example 4: Determine an equation of the tangent to the graph f(x) = 4/x at the point x=0.5.

$(0.5, \underline{8})$

$f(x) = \frac{4}{x}$

$= 4x^{-1}$

$f'(x) = -4x^{-2}$

$= \frac{-4}{x^2}$

$f'(0.5) = \frac{-4}{0.5^2}$

$= \frac{-4}{\left(\frac{1}{2}\right)^2}$

$= \frac{-4}{\frac{1}{4}} = -16$

$y = mx + b$

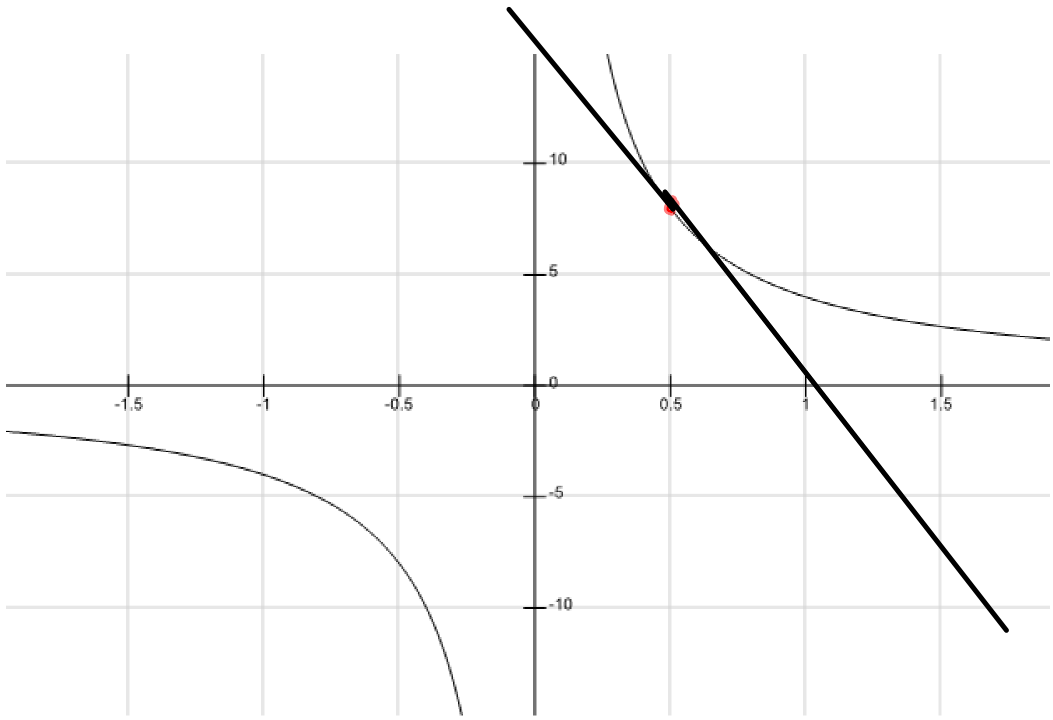
$y = -16x + b$

$8 = -16(0.5) + b$

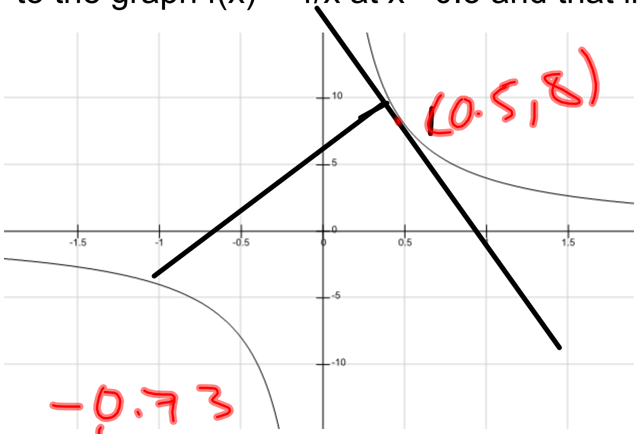
$8 = -8 + b$

$16 = b$

$\boxed{y = -16x + 16}$


10
5
0
-5
-10
-1.5
-1
-0.5
0
0.5
1
1.5

Example 5: Determine an equation of the line that is perpendicular to the tangent to the graph f(x) = 4/x at x= 0.5 and that intersects it at the point of tangency.



-0.73

$\frac{1}{0.73}$

$y = -\frac{16}{1}x + 16$

$y = +\frac{1}{16}x + b$

$8 = \frac{1}{16}(0.5) + b$

$8 = \frac{1}{32} + b$

$8 - \frac{1}{32} = b$

$\frac{256}{32} - \frac{1}{32} = b$

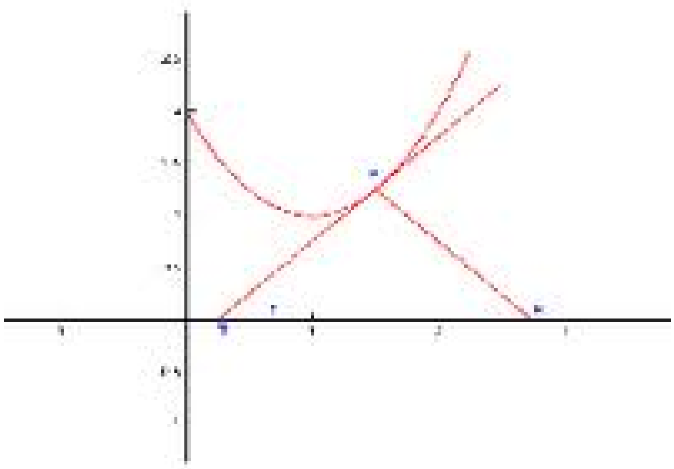
$\frac{255}{32} = b$

$y = \frac{1}{16}x + \frac{255}{32}$

$y = \frac{5}{1}x + 6$

$y = -\frac{1}{5}x + 7$

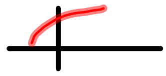
NORMAL - The normal to the graph of f(x) at P is the line that is perpendicular to the tangent at P.



The Existence of Derivatives

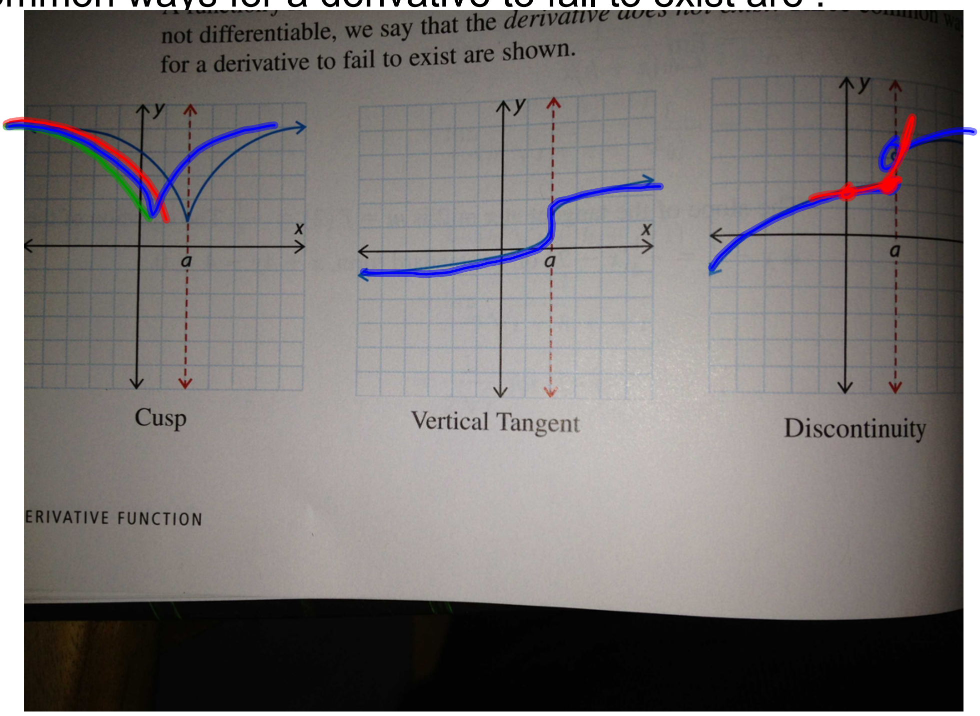
The function f(x) is said to be differentiable at a if f'(a) exists.

$f(x) = \sqrt{x+3}$



At points where f is not differentiable, we say the derivative does not exist.

Three common ways for a derivative to fail to exist are :



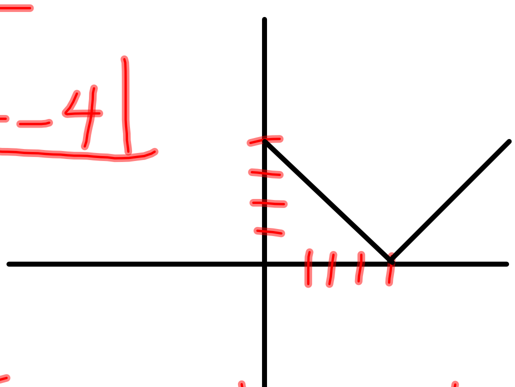
Example 6. Show that f(x) = |x−4| is not differentiable at x = 4

$$f'(x) = \lim_{h\to 0} \frac{f(4+h) - f(4)}{h}$$

$$= \lim_{h\to 0} \frac{|4+h-4| - |4-4|}{h}$$

$$= \lim_{h\to 0} \frac{|h| - 0}{h}$$

$$= \lim_{h\to 0} \frac{|h|}{h} \begin{cases} \lim_{h\to 0^-} \frac{-h}{h} = -1 \\ \lim_{h\to 0^+} \frac{+h}{h} = 1 \end{cases}$$



so limit does not exist so not differentiable @ x = 4

$f'(x)$

$$\frac{dy}{dx} = \lim_{h\to 0}$$

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