Chapter 5 Test – Rational Functions

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|  | R- | R+ | 1 | 2 | 3 | 4 |
| identify and describe some key features of the graphs of rational functions, and represent rational functions graphically; |  |  |  |  |  |  |
| solve problems involving polynomial and simple rational equations graphically and algebraically; |  |  |  |  |  |  |
| demonstrate an understanding of solving polynomial and simple rational inequalities. |  |  |  |  |  |  |
| demonstrate an understanding of average and instantaneous rate of change, and determine, numerically and graphically, and interpret the average rate of change of a function over a given interval and the instantaneous rate of change of a function at a given point; |  |  |  |  |  |  |

WHAT WILL THE TEST COVER???

**1. Understanding Rates of Change**

By the end of this course, students will:

1.1 gather, interpret, and describe information about real-world applications of rates of change, and recognize different ways of representing rates of change (e.g., in words, numerically, graphically, algebraically)

1.2 recognize that the rate of change for a function is a comparison of changes in the dependent variable to changes in the independent variable, and distinguish situations in which the rate of change is zero, constant, or changing by examining applications, including those arising from real-world situations (e.g., rate of change of the area of a circle as the radius increases, inflation rates, the rising trend in graduation rates among Aboriginal youth, speed of a cruising aircraft, speed of a cyclist climbing a hill, infection rates)

Sample problem: The population of bacteria in a sample is 250 000 at 1:00 p.m., 500 000 at 3:00 p.m., and 1 000 000 at 5:00 p.m. Compare methods used to calculate the change in the population and the rate of change in the population between 1:00 p.m. to 5:00 p.m. Is the rate of change constant? Explain your reasoning.

1.3 sketch a graph that represents a relationship involving rate of change, as described in words, and verify with technology (e.g., motion sensor) when possible

Sample problem: John rides his bicycle at a constant cruising speed along a flat road. He then decelerates (i.e., decreases speed) as he climbs a hill. At the top, he accelerates (i.e., increases speed) on a flat road back to his constant cruising speed, and he then accelerates down a hill. Finally, he comes to another hill and glides to a stop as he starts to climb. Sketch a graph of John's speed versus time and a graph of his distance travelled versus time.

1.4 calculate and interpret average rates of change of functions (e.g., linear, quadratic, exponential, sinusoidal) arising from real-world applications (e.g., in the natural, physical, and social sciences), given various representations of the functions (e.g., tables of values, graphs, equations)

Sample problem: Fluorine-20 is a radioactive substance that decays over time. At time 0, the mass of a sample of the substance is 20 g. The mass decreases to 10 g after 11 s, to 5 g after 22 s, and to 2.5 g after 33 s. Compare the average rate of change over the 33-s interval with the average rate of change over consecutive 11-s intervals.

1.5 recognize examples of instantaneous rates of change arising from real-world situations, and make connections between instantaneous rates of change and average rates of change (e.g., an average rate of change can be used to approximate an instantaneous rate of change)

Sample problem: In general, does the speedometer of a car measure instantaneous rate of change (i.e., instantaneous speed) or average rate of change (i.e., average speed)? Describe situations in which the instantaneous speed and the average speed would be the same.

1.6 determine, through investigation using various representations of relationships (e.g., tables of values, graphs, equations), approximate instantaneous rates of change arising from real-world applications (e.g., in the natural, physical, and social sciences) by using average rates of change and reducing the interval over which the average rate of change is determined

Sample problem: The distance, d metres, travelled by a falling object in t seconds is represented by d = 5t(2). When t = 3, the instantaneous speed of the object is 30 m/s. Compare the average speeds over different time intervals starting at t = 3 with the instantaneous speed when t = 3. Use your observations to select an interval that can be used to provide a good approximation of the instantaneous speed at t = 3.

1.7 make connections, through investigation, between the slope of a secant on the graph of a function (e.g., quadratic, exponential, sinusoidal) and the average rate of change of the function over an interval, and between the slope of the tangent to a point on the graph of a function and the instantaneous rate of change of the function at that point

Sample problem: Use tangents to investigate the behaviour of a function when the instantaneous rate of change is zero, positive, or negative.

1.8 determine, through investigation using a variety of tools and strategies (e.g., using a table of values to calculate slopes of secants or graphing secants and measuring their slopes with technology), the approximate slope of the tangent to a given point on the graph of a function (e.g., quadratic, exponential, sinusoidal) by using the slopes of secants through the given point (e.g., investigating the slopes of secants that approach the tangent at that point more and more closely), and make connections to average and instantaneous rates of change

1.9 solve problems involving average and instantaneous rates of change, including problems arising from real-world applications, by using numerical and graphical methods (e.g., by using graphing technology to graph a tangent and measure its slope)

Sample problem: The height, h metres, of a ball above the ground can be modelled by the function h(t) =– 5t(2) + 20t, where t is the time in seconds. Use average speeds to determine the approximate instantaneous speed at t = 3.

**2. Connecting Graphs and Equations of Rational Functions**

By the end of this course, students will:

2.1 determine, through investigation with and without technology, key features (i.e., vertical and horizontal asymptotes, domain and range, intercepts, positive/negative intervals, increasing/decreasing intervals) of the graphs of rational functions that are the reciprocals of linear and quadratic functions, and make connections between the algebraic and graphical representations of these rational functions [e.g., make connections between f(x) = 1/[x(2) – 4] and its graph by using graphing technology and by reasoning that there are vertical asymptotes at x = 2 and x =–2 and a horizontal asymptote at y = 0 and that the function maintains the same sign as f(x) = x(2) – 4]

Sample problem: Investigate, with technology, the key features of the graphs of families of rational functions of the form f(x) = 1/x + n, and f(x) = 1/[x(2) + n] where n is an integer, and make connections between the equations and key features of the graphs.

2.2 determine, through investigation with and without technology, key features (i.e., vertical and horizontal asymptotes, domain and range, intercepts, positive/negative intervals, increasing/decreasing intervals) of the graphs of rational functions that have linear expressions in the numerator and denominator [e.g., f(x) = 2x/[x– 3], h(x) = x – 2/(3x + 4)], and make connections between the algebraic and graphical representations of these rational functions

Sample problem: Investigate, using graphing technology, key features of the graphs of the family of rational functions of the form f(x) = 8x/(nx + 1) for n = 1, 2, 4, and 8, and make connections between the equations and the asymptotes.

2.3 sketch the graph of a simple rational function using its key features, given the algebraic representation of the function

**3. Solving Polynomial and Rational Equations**

By the end of this course, students will:

3.5 determine, through investigation using technology (e.g., graphing calculator, computer algebra systems), the connection between the real roots of a rational equation and the x-intercepts of the graph of the corresponding rational function, and describe this connection [e.g., the real root of the equation x – 2/x – 3 = 0 is 2, which is the x-intercept of the function f(x) = x – 2/x – 3; the equation 1/x – 3 = 0 has no real roots, and the function f(x) = 1/x – 3 does not intersect the x-axis]

3.6 solve simple rational equations in one variable algebraically, and verify solutions using technology (e.g., using computer algebra systems to determine the roots; using graphing technology to determine the x-intercepts of the graph of the corresponding rational function)

3.7 solve problems involving applications of polynomial and simple rational functions and equations [e.g., problems involving the factor theorem or remainder theorem, such as determining the values of k for which the function f(x) = x(3) + 6x(2) + kx – 4 gives the same remainder when divided by x – 1 and x + 2]

Sample problem: Use long division to express the given function f(x) = [x(2) + 3x – 5]/[x – 1] as the sum of a polynomial function and a rational function of the form A/x – 1 (where A is a constant), make a conjecture about the relationship between the given function and the polynomial function for very large positive and negative x-values, and verify your conjecture using graphing technology.

**4. Solving Inequalities**

By the end of this course, students will:

4.1 explain, for polynomial and simple rational functions, the difference between the solution to an equation in one variable and the solution to an inequality in one variable, and demonstrate that given solutions satisfy an inequality (e.g., demonstrate numerically and graphically that the solution to 1/x + 1 [less than symbol] 5 is x [less than symbol] –1 or x [greater than symbol] – 4/5);