$$4^{2x} = 5^{2x-1}$$

$$\log 4^{2x} = \log 5^{2x-1}$$

$$2x\log 4 = (2x-1)\log 5$$

$$2x\log 4 = 2x\log 5 - \log 5$$

$$2x\log 4 - 2x\log 5 = -\log 5$$

$$x(2\log 4 - 2\log 5) = -\log 5$$

$$2x = \frac{-\log 5}{\cancel{2}\log 4 - \cancel{2}\log 5}$$

$$x = (\qquad)/2$$

$$= 3.6$$

# Solving Logarithmic Equations

8.6

**Example 1:** Solve.

$\log_5(2x-1) = \log_5 47$

$2x-1 = 47$

$2x = 48$

$x = 24$

$2^{x+3} = 2^8$

$x+3 = 8$

$x = 5$

**Example 2:** Solve $\log_x 6 = -1/2$

$$\log_x 6 = -\frac{1}{2}$$

$$-\frac{1}{2} \cdot -\frac{2}{1}$$

$$x^{-\frac{1}{2} \cdot -\frac{2}{1}} = 6^{-\frac{2}{1}}$$

$$x = \frac{1}{6^2}$$

$$x^1 = x = \frac{1}{36}$$

Now you try: $\log_x 0.04 = -2$

$$\log_5 0.04 = \frac{\log 0.04}{\log 5}$$

$$x^{-\frac{2}{1}} = 0.04$$

$$x^{-\cancel{2} \cdot \frac{1}{-\cancel{2}}} = 0.04^{\frac{1}{-2}}$$

$$x = 5$$

**Example 3:** Solve

$\log_3 25x - \log_3 5 = \log_3 20$

$\log_3 25x = \log_3 20 + \log_3 5$

$\log_3 25x = \log_3 100$

$25x = 100$

$x = \frac{100}{25}$

$= 4$

**Example 4:** $\log x + \log x^3 = 20$

$\log x \cdot x^3 = 20$

$\log x^4 = 20$

$4 \log x = 20$

$\log_{10} x = 5$

$10^5 = x$

$100000 = x$

Now you try:

$\log x + \log x^2 = 12$

$\log(x \cdot x^2) = 12$

$\log x^3 = 12$

$3 \log x = 12$

$\log_{10} x = 4$

$10^4 = x$

$10000 = x$

$\log 7$

$\log_2 7$

**Example 5:** Solve $\log_7(x+1) + \log_7(x-5) = 1$

$$\log_7((x+1)(x-5)) = 1$$

$$\log_7(x^2-5x+x-5) = 1$$

$$\log_7(x^2-4x-5) = 1$$

Now you try

$\log_2(x+3)+\log_2(x-3) = 4$

$$7^1 = x^2-4x-5$$

$$7 = x^2-4x-5$$

$$0 = x^2-4x-12$$

$$0 = (x-6)(x+2)$$

$$x-6 = 0$$
$$x = 6$$

or

$$x+2 = 0$$
$$x = -2$$

Now you try

$\log_2(x+3)+\log_2(x-3) = 4$

$$\log_2((x+3)(x-3)) = 4$$

$$2^4 = (x+3)(x-3)$$

$$16 = x^2-9$$

$$0 = x^2-25$$

$$0 = (x-5)(x+5)$$

$$x = 5, -5$$

**Example 6:** Solve $\log_6 x + \log_6(x-5) = 2$. Check for inadmissible roots.

$$
\begin{aligned}
\log_6 x(x-5) &= 2 \\
6^2 &= x(x-5) \\
36 &= x^2 - 5x \\
0 &= x^2 - 5x - 36 \\
0 &= (x-9)(x+4) \\
x &= 9 \text{ or } -4. \quad \text{Which is inadmissible????}
\end{aligned}
$$

**Example 7:** The Richter scale is used to compare the intensities of earthquakes. The Richter scale magnitude,R, of an earthquake is determined using

$$R=\log(a/T) + B,$$

where a is the amplitude of the vertical ground motion in microns (μ) , T is the period of the seismic wave in seconds, and B is a factor that accounts for the weakening of the seismic waves.

An earthquake measures 5.5 on the Richter scale, and the period of the seismic wave was 1.8 s. If B equals 3.2, what was the amplitude, a, of the vertical ground motion?

Solution

$$R = \log (a/T) + B$$
$$5.5 = \log(a/1.8) +3.2$$
$$5.5-3.2=\log(a/1.8)$$
$$2.3 = \log(a/1.8)$$
$$10^{2.3} = a/1.8$$
$$1.8\times10^{2.3} = a$$

The amplitude was about 359.1 μ.

**Example 8:**

The loudness, L, of a sound in decibels, (dB) can be calculated using
$L = 10 \log (I/I^0)$,
where I is the intensity of sound in watts per square metre ($W/m^2$) and $I^0 = 10^{-12}$ $W/m^2$.
Determine the intensity of a baby screaming if the noise level is 100dB.

$$100 = 10 \log (I/10^{-12})$$

$$100/10 = \log (I/10^{-12})$$
$$10 = \log (I/10^{-12})$$
$$10^{10} = I/10^{-12}$$
$$10^{10} \times 10^{-12} = I$$
$$10^{-2} = I$$

## Now you try : pg 492 #9

$L = 10 \log (I/I^0)$,
where I is the intensity of sound in watts per square metre ($W/m^2$) and $I^0 = 10^{-12}$ $W/m^2$.

a) A teacher is speaking to a class. Determine the intensity of the teacher's voice if the sound level is 50dB

Homework: pg 491-492
#1def
#2def
#3
#4cdef
#5def
#7
#9b
#12
#13
#14
#15
#16