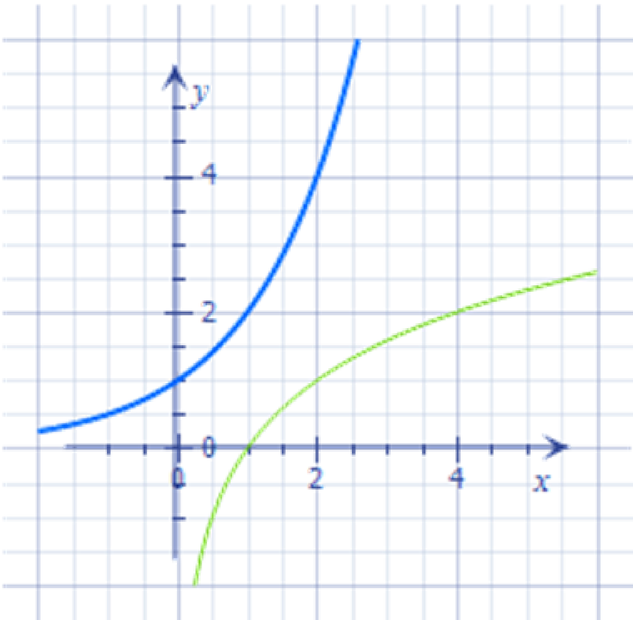
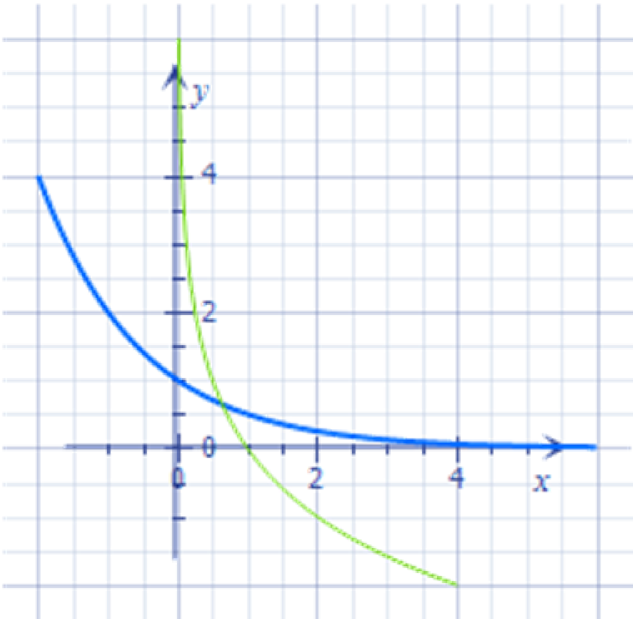
Graphing Logarithms

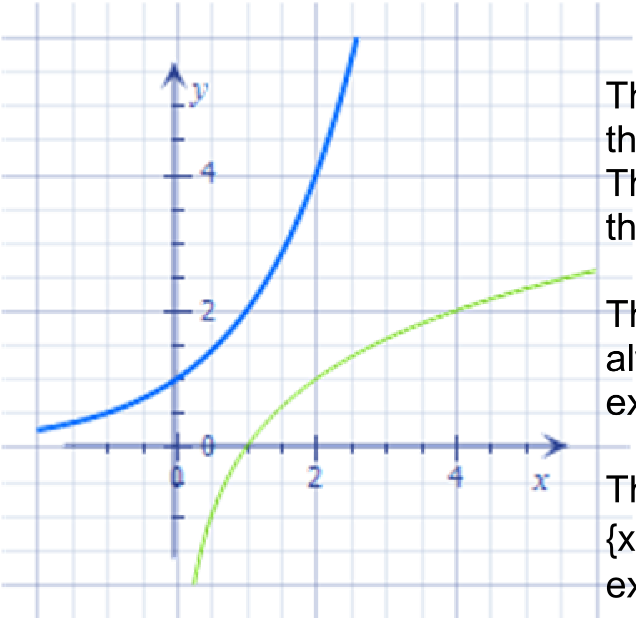
Important! The general shape of the graph a logarithmic functions depends on the value of the base.

When a>0, the exponential is an increasing function and so in the logarithmic function.



When 0<a<1, the exponential is decreasing and so is the logarithmic function.

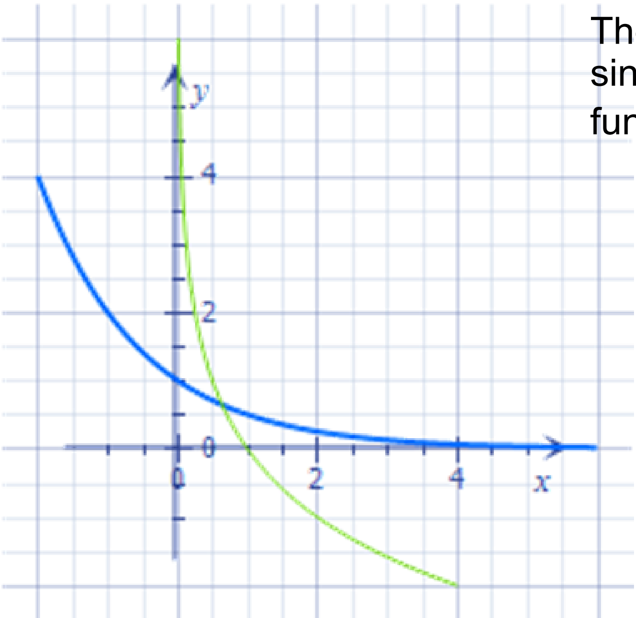




The y-axis is the vertical asymptote for the log function.
The x-axis is the horizontal asymptote for the exponential function.

The x-intercept for the log function is always 1, while the y-intercept for the exponential function is 1.

The domain of the log function is $\{x \in R | x>0\}$, since the range of the exponential is $\{x \in R | x>0\}$



The range of the log function is $\{y \in R\}$ since the domain of the exponential function is $\{x \in R\}$

Transforming Logarithmic Functions

A logarithmic function of the form $f(x) = a \log_{10}(k(x-d)) + c$ can be graphed by applying the appropriate transformations for the parent function, $f(x) = \log_{10}x$

To graph a transformed log function, apply the stretches/compressions/reflectioins given by a and k first. Then apply the vertical and horizontal translations given by c and d.

$$f(x) = a \log_{10}(k(x-d)) + c$$

$\lvert a \rvert$	gives the vertical stretch/compression factor. If a<0, there is also a reflection in the x-axis Multiply all y by a
$\lvert 1/k \rvert$	gives the horizontal stretch/compression factor. If k<0 there is also a reflection in the y- axis. Divide all x by k
d	gives the horizontal translation.
c	gives the vertical translation.

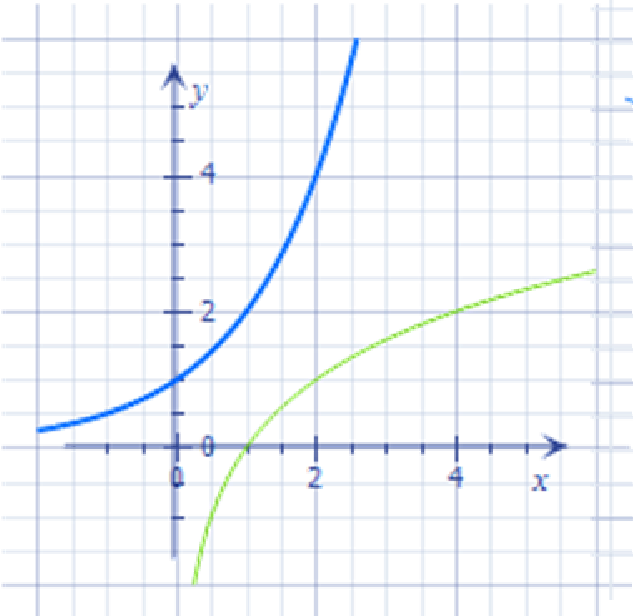
$f(x) = a \log_{10}(k(x-d)) + c$

Since "d" gives the horizontal translation, it translates the vertical asymptote.

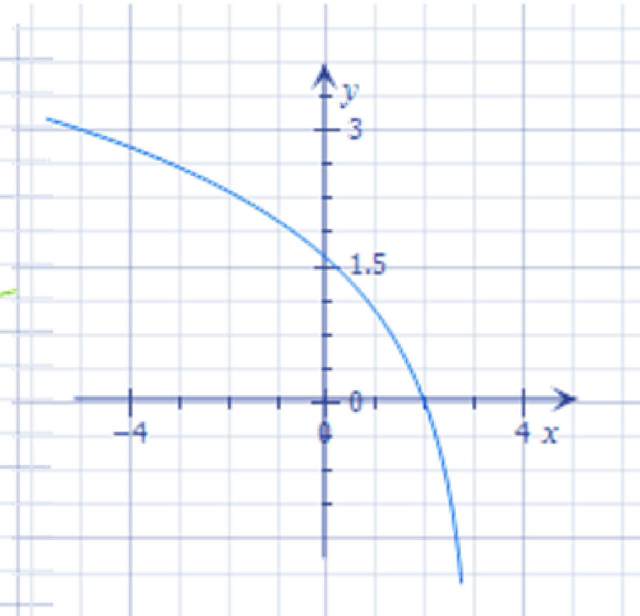
Recall that with an exponential function, there is a horizontal asymptote which "c" translates up or down.

Since a log function has no horizontal asymptote, but instead a vertical asymptote, it is the "d" that translates it left or right.

The domain of a transformed logarithmic function depends on where the vertical asymptote is located and whether the function is to the left or right of the vertical asymptote.



$y=\log_2 x$



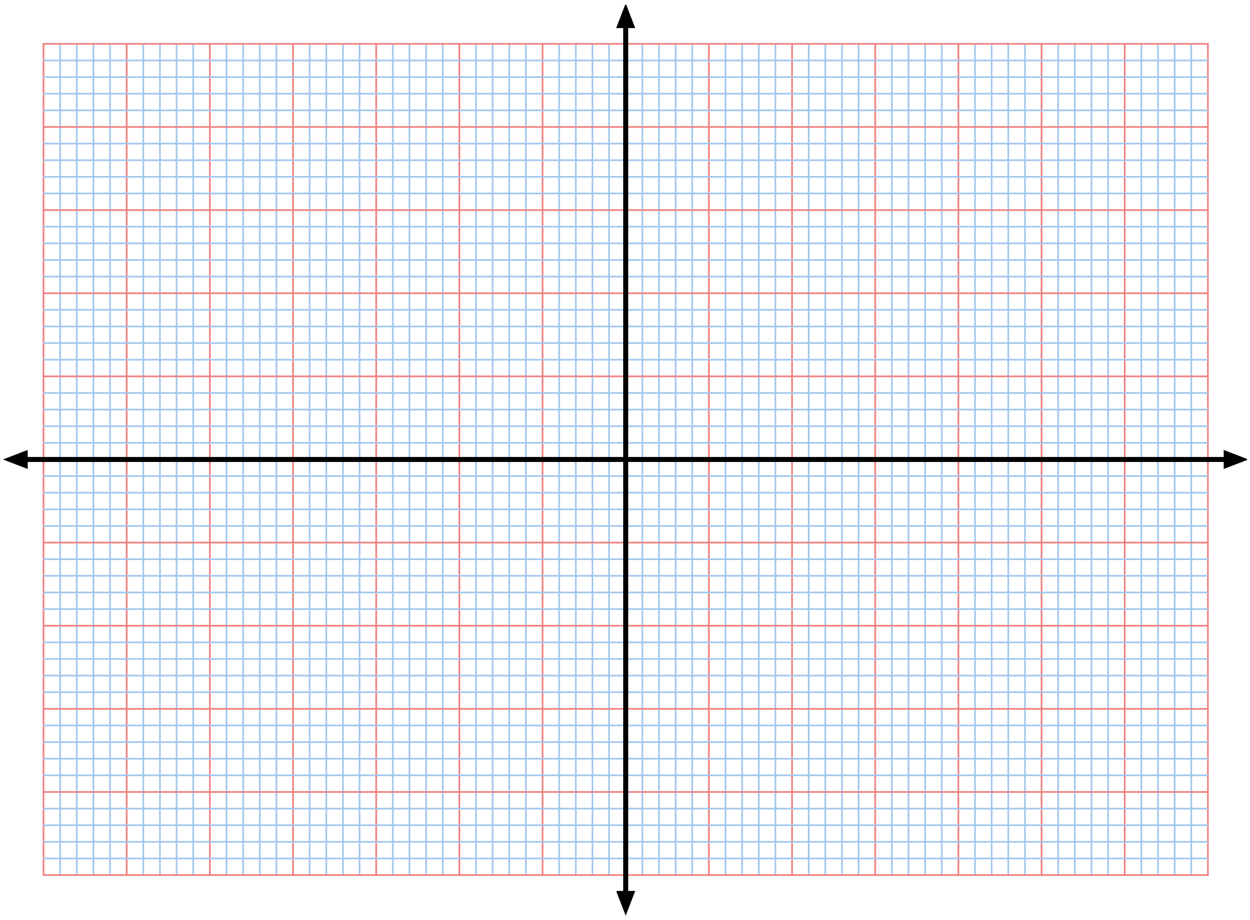
$y=\log_2(-(x-3))$

Create a table for the parent function, $f(x) = \log_{10}x$

x	$y=\log_{10}x$
1/10	-1
1	0
10	1
32	1.5

Example 1: Use transformations to sketch $-2\log_{10}(x-4)$

x	$y=1\log_{10}x$	$-2\log_{10}x$	$-2\log_{10}(x-4)$
1/10	-1		
1	0		
10	1		
32	1.5		



Domain: Range:

Now you try: $g(x) = 2 \log_{10}[-2(x+2)]$

Example 2: The log function y =$\log_{10}x$ has be vertically compressed by a factor of 2/3, horizontally compressed by a factor of 4 and then reflected in the y-axis. It has been horizontally translated so that the vertical asymptote is x=-5 and then vertically translated 2 units up. Write an equation of the transformed function. State the domain and range.

Now you try. State the equation of the function that results from a reflection in the x-axis, horizontal translation 4 units right, vertical stretch of factor 5 and horizontal compression by factor 1/9.

(Now back to section 8.5)

Example 3: Solve $2^{x+2} - 2^{x} = 24$

$2^{x} \cdot 2^{2} - 2^{x} = 24$

$2^{x}(2^{2} - 1) = 24$

$2^{x} = \frac{24}{2^{2} - 1} = \frac{24}{3} = 8$

$2^{x} = 8$

$x = 3$

$2^{x} = 8$

$\log 2^{x} = \log 8$

$x \log 2 = \log 8$

$x = \frac{\log 8}{\log 2}$

You try:

$$4^{x+3} - 4^{x} = 63$$

(Now back to section 8.5)

Example 3: Solve $2^{x+2} - 2^x = 24$

You try:

$$4^{x+3} - 4^x = 63$$

$x = \log 8$

$x = \frac{\log 8}{\log 2}$

Solution:

$4^{x+3} - 4^{x} = 63$

Example 4: Solve $2^{x+1} = 3^{x-1}$

$$2^{x+1} = 3^{x-1}$$

$$\log 2^{x+1} = \log 3^{x-1}$$

$$2^{4.42+1} = 3^{4.42-1}$$

$$2^{5.42} = 3^{3.42}$$

$$2^{x+1} = 3^{x-1}$$

$$\log 2^{x+1} = \log 3^{x-1}$$

$$(x+1)\log 2 = (x-1)\log 3$$

$$x\log 2 + \log 2 = x\log 3 - \log 3$$

$$x\log 2 - x\log 3 = -\log 3 - \log 2$$

$$x(\log 2 - \log 3) = -\log 3 - \log 2$$

$$x = \frac{-\log 3 - \log 2}{\log 2 - \log 3} = 4.42$$

Now you try: $4^{2x} = 5^{2x-1}$

Example 4: Solve $2^{x+1} = 3^{x-1}$

$$2^{x+1} = 3^{x-1}$$

$$\log 2^{x+1} = \log 3^{x-1}$$

Now you try: $4^{2x}=5^{2x-1}$

$x(\log 2 - \log \cdot$

$x = \dfrac{-\log 3 - \log 2}{\log 2 - \log 3}$

Now you try: $4^{2x}=5^{2x-1}$

Homework:
pg457-458

#1,3,4iii,v,vi , 5def, 6,7,8

Pg 486 #8,10

pg 475
#6c
#6a

$\log_6 6\sqrt{6}$ (2 inside)

$= \log_6 \sqrt{6 \cdot 6 \cdot 6}$

$= \log_6 216^{1/2}$

$= \frac{1}{2} \log_6 216$

$\sqrt[3]{\ } = {}^{1/3}$

$= \frac{1}{2}(3)$

$= \frac{3}{2}$

d)

$$\log_2 \sqrt{36} - \log_2 \sqrt{72}$$

$$= \log_2 36^{1/2} - \log_2 72^{1/2}$$

$$= \frac{1}{2}\log_2 36 - \frac{1}{2}\log_2 72$$

$$= \frac{1}{2}\left(\log_2 36 - \log_2 72\right)$$

$$= \frac{1}{2}\log_2\left(\frac{36}{72}\right) = \frac{1}{2}\log_2\left(\frac{1}{2}\right)$$

$$= \frac{1}{2}(-1)$$

$$= -\frac{1}{2}$$

$$2^{-5} = \frac{1}{2^5}$$

$$2^{-3} = \frac{1}{2^3}$$

$$2^{-2} = \frac{1}{2^2}$$

$$2^{-1} = \frac{1}{2^1} = \frac{1}{2}$$

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#(6b)

interest / compounding period

$$A = P(1+i)^n$$

# of compounding periods

$$5000 = 1000\left(1+\frac{0.12}{12}\right)^n$$

$$\frac{5000}{1000} = \left(1+\frac{0.12}{12}\right)^n$$

$$5 = \left(1+\frac{0.12}{12}\right)^n$$

$$\log 5 = \log\left(1+\frac{0.12}{12}\right)^n$$

$$\log 5 = n \log\left(1+\frac{0.12}{12}\right)$$

$$\frac{\log 5}{\log\left(1+\frac{0.12}{12}\right)} = n$$

n is the # of compounding periods.

$$162 \doteq n$$

$$162 \text{ months} = \frac{162}{12} = 13.5 \text{ years}$$