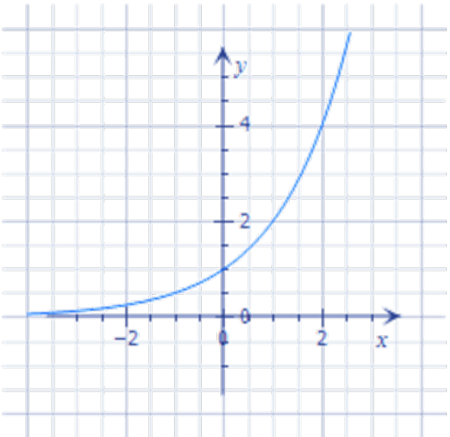
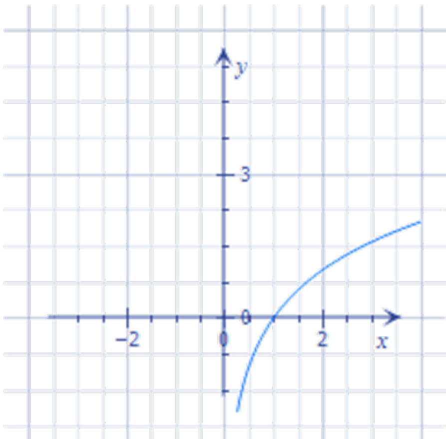
Logarithmic Functions

What is a logarithmic Function?   It is the INVERSE of an exponential function.

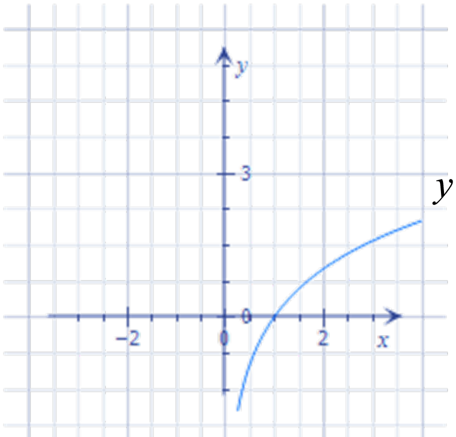


$$y = 2^x$$



$$y = \log_2 x$$

We read $\log_a x$ as "the logarithm of x to base a"



$y = \log_2 x$

y as a function of x

Or if you think of x as a function of y , this curve can be labelled

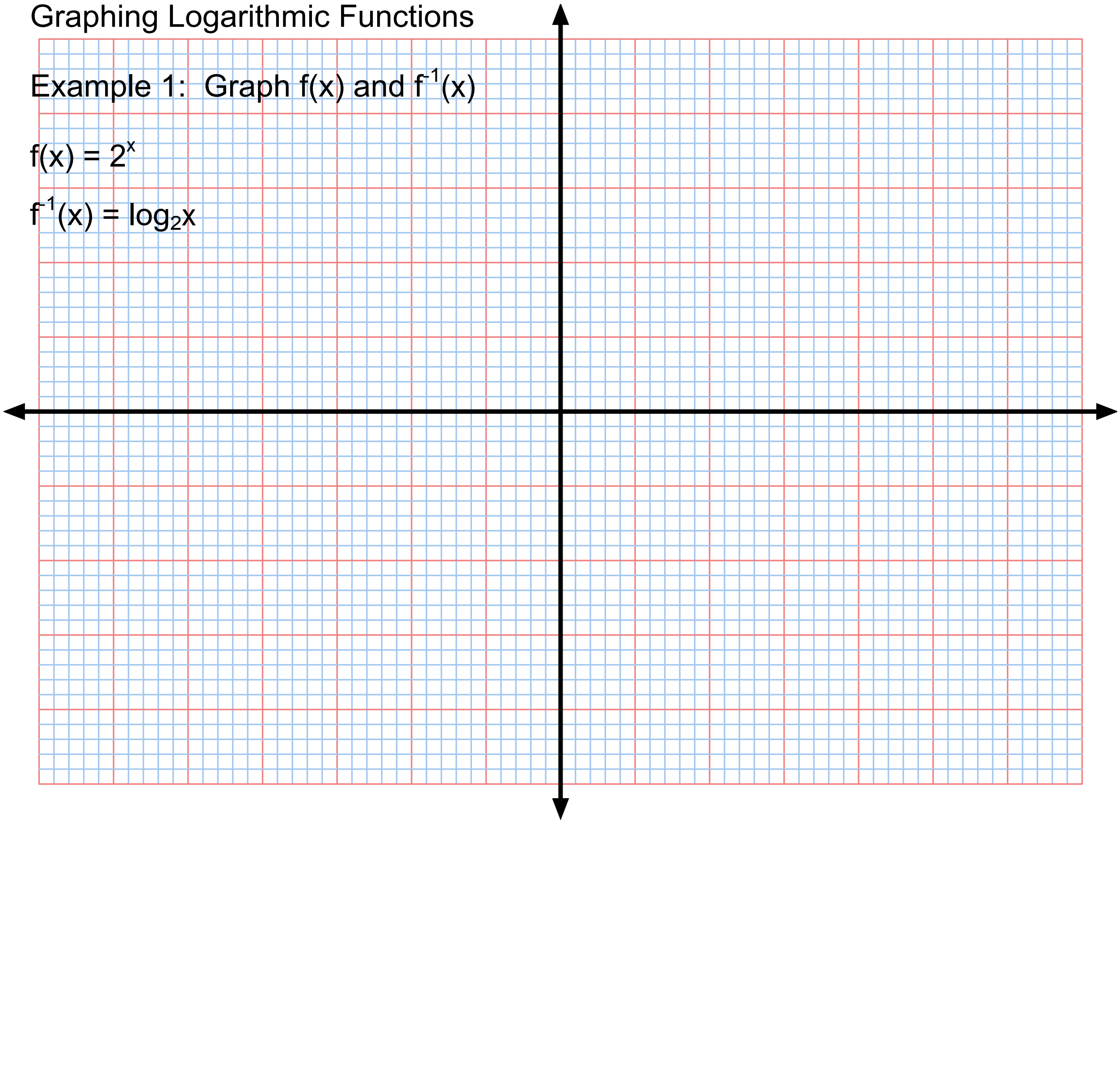
$x=2^y$

Graphing Logarithmic Functions

Example 1: Graph $f(x)$ and $f^{-1}(x)$

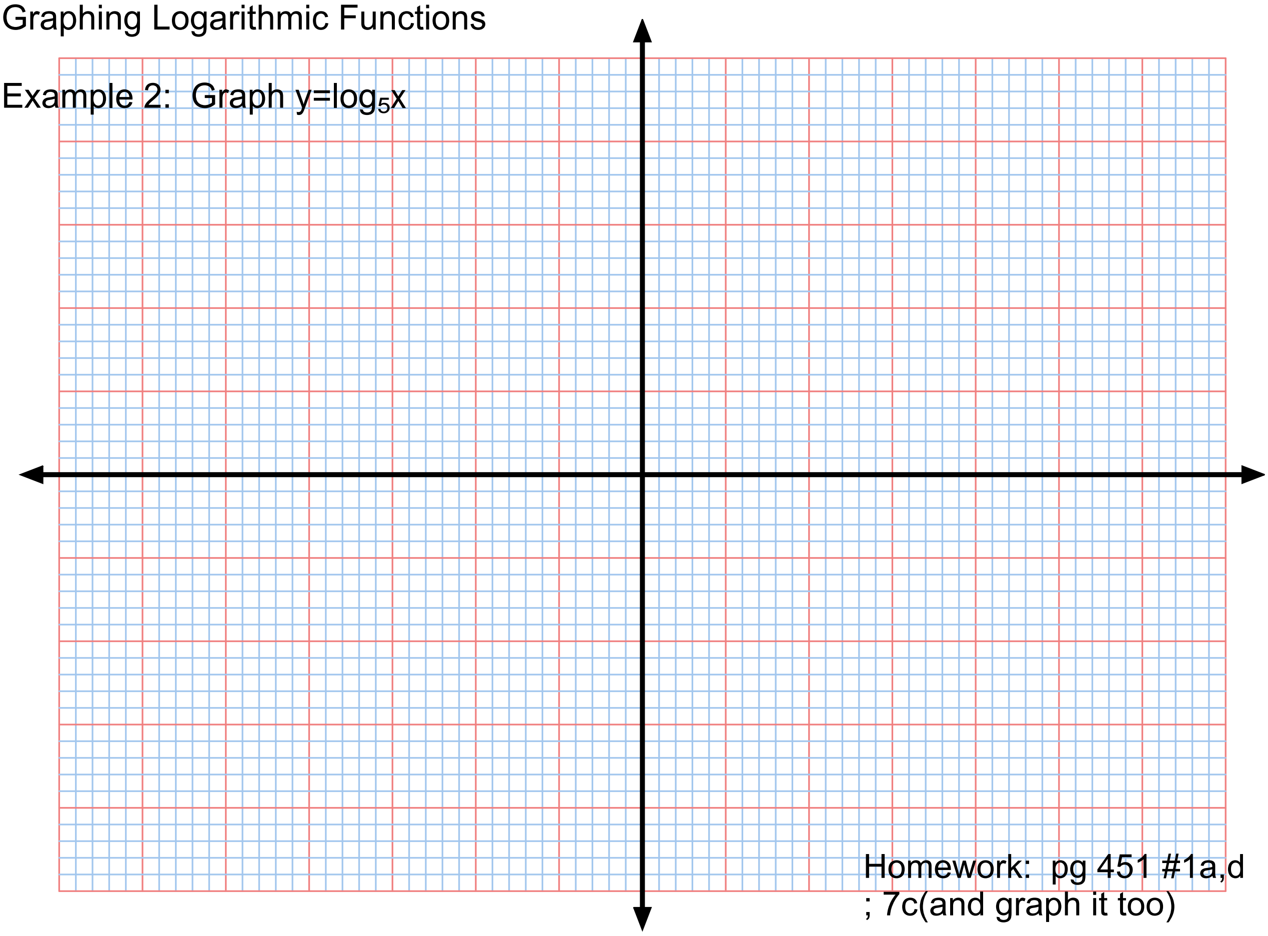
$f(x) = 2^x$

$f^{-1}(x) = \log_2 x$



Graphing Logarithmic Functions

Example 2: Graph $y=\log_5 x$



Homework: pg 451 #1a,d ; 7c(and graph it too)

Solving Log Equations

$\log_5 5^2 = 2$

$2\log_5 5$

$\log_a 1 = 0$

$\log_a a^x = x$

$a^{\log_a x} = x$

$a^0 = 1$

$\log_a a = x,$

$a^x = a$

$x = 1$.

$\log_a a^x = x \log_a a$

$= x(1)$

$= x$

$2^3 = 8$

$\log_2 8 = 3$

$a^{(\log_a x)} = y = x$

$\log_a y = \log_a x$

$y = x$

LAWS OF LOGARITHMS

Because logarithms are exponents, the laws of exponents give rise to corresponding laws of logarithms.

PRODUCT LAW

$\log_a(MN) = \log_a M + \log_a N$

QUOTIENT LAW

$\log_a(M/N) = \log_a M - \log_a N$

$\log_a\left(\frac{5}{7}\right) = \log_a 5 - \log_a 7$

POWER LAW

$\log_a M^n = n \log_a M$

Example 3: Simplify

a) $\log_3 6 + \log_3 4.5$

$= \log_3 (6 \times 4.5)$

$= \log_3 (27)$

$3^x = 27$

$x = 3$

b) $\log_2 48 - \log_2 3$

$= \log_2 (48/3)$

$= \log_2 16$

$= y$

$2^y = 16$

$y = 4$

Example 4: Evaluate $\log_3(\sqrt[5]{27/2187})$

$$\log_3 \sqrt[5]{\frac{1}{27}}$$

$$= \log_3 \left(\frac{1}{27}\right)^{1/5}$$

$$= \frac{1}{5} \log_3 \left(\frac{1}{27}\right) \qquad 3^{-3} = \frac{1}{27}$$

$$= \frac{1}{5}(-3)$$

$$= -3/5$$

Example 5: Use the laws of logs to express each side of the equation as a single log. Then compare both sides to solve.

① Write as a single log part of the equation

$\log_3 x = 2\log_3 10 - \log_3 25$

$= \log_3 10^2 - \log_3 25$

$= \log_3 100 - \log_3 25$

$= \log_3 \left(\frac{100}{25}\right)$

$\log_a b^x = x\log_a b$

$\log_3 x = \log_3 4$

$x = 4$

Homework: pg 475 #
4,6,10,12

Interesting!!! Now we will use the calculator. The calculator assumes base 10.

Evaluate a) $\log_{10} 1000$

$= \frac{\log 1000}{\log 10}$

$= 3$

` b) $\log_{10} 0.01$

$= \frac{\log 128}{\log 2}$

c) $\log_2 128$

$$\log_a b = \frac{\log b}{\log a}$$

$$\log_a b = \frac{\log b}{\log a}$$

Example 6: Evaluate $\log_7 54$

$$= \frac{\log 54}{\log 7}$$

$$= 2.05$$

$$\log_7 54 = 2.05$$

$$7^{2.05} \doteq 54$$

Example 7: Solve for x, a) if $3^x = 187$

$$3^x = 187$$

Take log of both sides

$\log 3^x = \log 187$
$x \log 3 = \log 187$
$x = \log 187 / \log 3$

$$\log 3^x = \log 187$$

$$x \cdot \log 3 = \log 187$$

$$x = \frac{\log 187}{\log 3}$$

$$= 4.76$$

b) if $5^x = 7893$

Example 7: Solve for x, a) if $3^x = 187$

Take log of both sides

$\log 3^x = \log 187$
$x \log 3 = \log 187$
$x = \log 187/\log 3$

b) if $5^x = 7893$

$\log 5^x = \log 7893$

$x \cdot \log 5 = \log 7893$

$x = \log 7893/\log 5 = 5.57$

$= 4.16$

Recall:

Half -Life Formula

$$f(t) = a\,(.5)^{t/h}$$

Doubling Time Formula

$$f(t) = a(2)^{t/d}$$

Solving Exponential Equations

Example 8: The half -life of radium -226 is 1620 years. Starting with a sample of 120mg, after how many years will there be only 40mg left?

$R(t) = 120\,(.5)^{t/1620}$

Solution:

$$40 = 120(.5)^{t/1620}$$

$$40/120 = (.5)^{t/1620}$$

$$1/3 = (.5)^{t/1620}$$

take log of both sides

$$\log\left(\tfrac{1}{3}\right) = \log(0.5)^{t/1620}$$

$$\log\left(\tfrac{1}{3}\right) = \frac{t}{1620}\cdot\log(0.5)$$

$$\log\left(\tfrac{1}{3}\right)\times 1620 = t\cdot\log 0.5$$

$$\frac{\log\left(\tfrac{1}{3}\right)\times 1620}{\log 0.5} = t$$

$$2567.6 = t$$

Example 9: Ahlam works in a lab. The lab received a shipment of 200g of radioactive radon, and 16 days later, 12.5g of the radon remained. What is the half-life?

$$F(t) = 200(0.5)^{16/h}$$

$$12.5 = 200(0.5)^{16/h}$$

$$\frac{12.5}{200} = 0.5^{16/h}$$

$$\log\left(\frac{12.5}{200}\right) = \log 0.5^{16/h}$$

$$\log\left(\frac{12.5}{200}\right)h = \frac{16}{\cancel{h}} \log 0.5$$

$$h = 16 \log 0.5 / \log\left(\frac{12.5}{200}\right)$$

$$= 4$$

Homework: pg 485 #1a, 2a,b,4,6,7,