7. 6 Solving Quadratic Trigonometric Equations

Three Methods for Solving Quadratic Functions:

$(4x-3)(x+2) = 0$

$4x-3=0$ $\quad x+2=0$

or $x=\frac{3}{4}$ $\quad x=-2$

$(4\cos x-3)(\cos x+2)=0$

$4\cos x-3=0$ or $\cos x+2=0$

$x^2 - 3x -4 = 0$

$(x-4)(x+1)=0$

or $x-4=0$ $\quad x+1=0$

$\sin^2 x - 3\sin x - 4 = 0$

$(\sin x - 4)(\sin x+1)=0$

$4x^2 - 17x -2 = 0$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{17 \pm \sqrt{17^2-4(4)(2)}}{2(4)}$$

$$x = \frac{3 \pm \sqrt{17}}{2}$$

$$\sin x = \frac{3 \pm \sqrt{17}}{2}$$

Example 1: Solve for x , $0 \leq x \leq 2\pi$

$$(2\cos x - 1)(2\sin x + \sqrt{3}) = 0$$

$2\cos x - 1 = 0$ or $2\sin x + \sqrt{3} = 0$

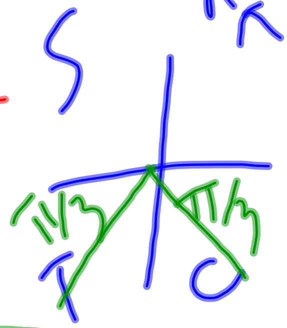
$2\cos x = 1$ $\quad$ $2\sin x = -\sqrt{3}$

$\cos x = \frac{1}{2}$ $\quad$ $\sin x = -\frac{\sqrt{3}}{2}$

$\cos x_R = \frac{1}{2}$ $\quad$ $\sin x_R = \frac{\sqrt{3}}{2}$

$x_R = \frac{\pi}{3}$ $\quad$ $x_R = \frac{\pi}{3}$

$x = \frac{\pi}{3}$ $\quad$ $\frac{5\pi}{3}$



$x = \frac{4\pi}{3}$

$x = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{4\pi}{3}$

Example 2:

Solve for x , $0 \leq x \leq 2\pi$

$$3\tan^2 x = 1$$ let $\tan x = x$

$x^2 = 4$

$x = \pm\sqrt{4}$

$= \pm 2$

$3x^2 = 1$

$x^2 = \frac{1}{3}$

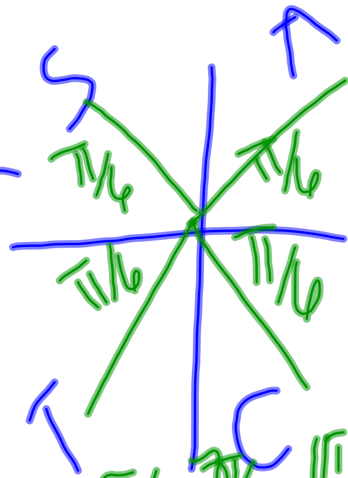
$x = \pm\sqrt{\frac{1}{3}}$

$\tan x = \pm\sqrt{\frac{1}{3}}$

$\tan x = \pm\frac{1}{\sqrt{3}}$

$\tan x_R = \frac{1}{\sqrt{3}}$

$x_R = \pi/6$



$x = \pi/6, 5\pi/6, 7\pi/6, 11\pi/6$

Example 3:

Solve each equation for x in the interval $0 \le x \le 2\pi$

$\cos^2 x - 2\cos x = -1$

$12\sin^2 x - \sin x - 1 = 0$

let $\sin x = y$

$12y^2 - y - 1 = 0$

$12 \times -1 = -12$

$= -1$

$+$

$(12y^2 - 4y) + (3y - 1) = 0$

$4y(3y-1) + 1(3y-1) = 0$

$(4y+1)(3y-1) = 0$

$4y+1 = 0$, $y = -1/4$ $\quad$ $y = \frac{1}{3}$

$12y^2 - y - 1 = 0$

$y = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{1 \pm \sqrt{1-4(12)(-1)}}{2(12)}$

$= \frac{1 \pm \sqrt{49}}{2(12)}$

$= \frac{1 \pm 7}{24}$

$= \frac{8}{24}, \frac{-6}{24}$

$= \frac{1}{3}, -\frac{1}{4}$

$y = \frac{1}{3}$

$\sin x = 1/3$

$x_R = \sin^{-1}\left(\frac{1}{3}\right)$

$x_R = 0.34$

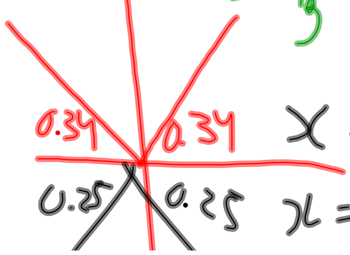
$R = 0.34$, $\pi - 0.34$

$= 0.34$, 2.8

$y = \frac{-1}{4}$

$\sin x = -\frac{1}{4}$

$x_R = \sin\left(\frac{1}{4}\right)$



$x = \pi + 0.25 = 3.39$

$x = 2\pi - 0.25 = 6.03$

Example 3:

Solve each equation for x in the interval $0 \leq x \leq 2\pi$

$cos^2 x - 2cos x = -1$

$12sin^2 x - sin x - 1 = 0$

$\cos^2 x - 2\cos x = -1$

let $\cos x = y$

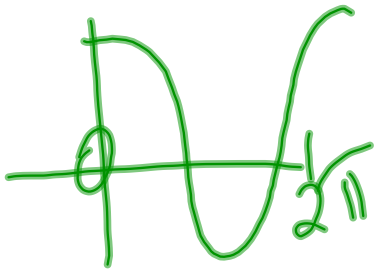
$y^2 - 2y = -1$ 1

$y^2 - 2y + 1 = 0$ 2

$(y-1)(y-1) = 0$ 3

$y - 1 = 0$ 4

$y = 1$ 5



$\cos x = 1$ 6 7

$x = 0, 2\pi$

$\cos x = 1$

$x = \cos^{-1}(1)$

$= 0$

Example 4:

Solve for x , $0 \le x \le 2\pi$

$3\sec^2 x - 4 = 0$ $\quad$ let $\cancel{x = \sec x}$

let $y = \sec x$

$$3y^2 - 4 = 0$$

$$3y^2 = 4$$

$$y^2 = \frac{4}{3}$$

$$y = \pm\sqrt{\frac{4}{3}}$$

$$y = \pm\frac{2}{\sqrt{3}}$$

$$\sec x = \pm\frac{2}{\sqrt{3}}$$

$$\cos x = \pm\frac{\sqrt{3}}{2}$$

$$x_R - \pi/6$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

Example 5:

For each equation, use a trig identity to create a quadratic equation. Then solve the equation for x in the interval $[0,2\pi]$

$4cot^2 - 1 + csc^2x = 0$

$cosx + cos2x = \cancel{9}$

$\cos 2\theta = 2\cos^2\theta - 1$

$$4\cot^2 x - 1 + \csc^2 x = 0$$

$$4\cot^2 x \cancel{-1} + (\cancel{1} + \cot^2 x) = 0$$

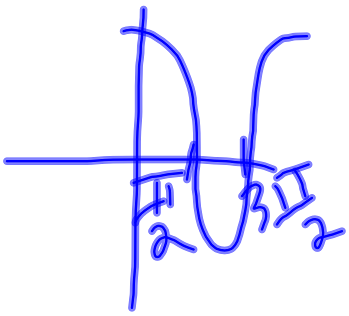
$$5\cot^2 x = 0$$

let $y = \cot x$

$$\frac{5y^2}{\cancel{5}} = \frac{0}{5}$$

$$y^2 = 0$$

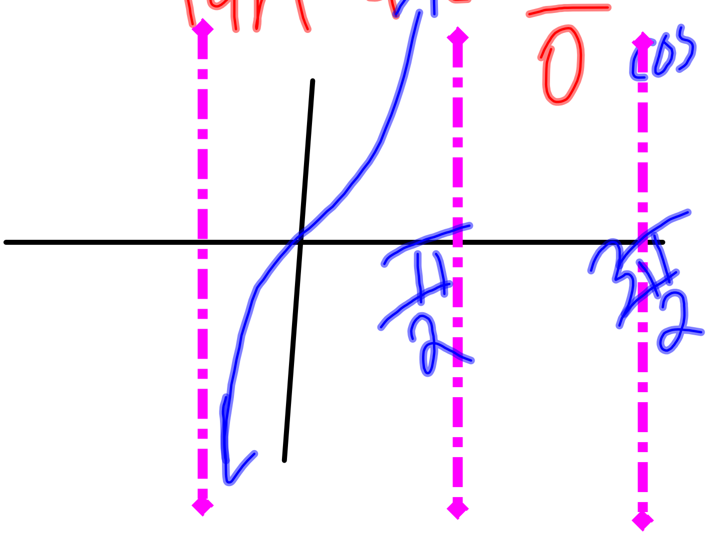
$$y = 0$$



$$\cot x = \frac{0}{1}$$

$$\tan x = \frac{1}{0} \quad \frac{\sin}{\cos}$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$



Example 5:

For each equation, use a trig identity to create a quadratic equation. Then solve the equation for x in the interval $[0,2\pi]$

$4cot^2 - 1 + csc^2x = 0$

$cosx + cos2x = 9$

$\cos 2\theta = 2\cos^2\theta - 1$

$\cos x + \cos 2x = 9$

$\cos x + 2\cos^2 x - 1 = 9$

let $y = \cos x$

$y + 2y^2 - 1 - 9 = 0$

$2x = 20$

$2y^2 + y - 10 = 0$

$(2y^2 - 4y) + (5y - 10) = 0$

$2y(y-2) + 5(y-2) = 0$

$(2y+5)(y-2) = 0$

$\frac{a}{h}$

$y = \frac{-5}{2}$ or $y = +2$

$\cos x = \frac{-5}{2}$ $\Rightarrow$ impossible

$\cos x = \frac{2}{1}$ $\Rightarrow$ impossible

Example 6:

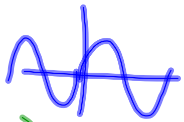
Malus's Law states that $I = I_o cos^2 \theta$

Use the law to determine the angle between polarizer A and polarizer B that will reduce the light intensity by 94%.

Example 7:

Solve the equation $8\sin^2 x - 8\sin x + 1 = 0$, for $-2\pi \le x \le 0$

Let $y = \sin x$



$$8y^2 - 8y + 1 = 0$$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{8 \pm \sqrt{64 - 4(8)(1)}}{2(8)}$$

$$= \frac{8 \pm \sqrt{32}}{16}$$

$y = 0.853$ or 0.146

$$= \frac{8 \pm \sqrt{16 \cdot 2}}{16}$$

$\sin x = 0.853$

$$= \frac{8 \pm 4\sqrt{2}}{16}$$

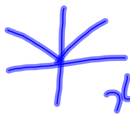
$\sin x = 0.146$

$$= \frac{2 \pm \sqrt{2}}{4}$$

$x_R = \sin(0.146)$

$x = 0.146$ ✓

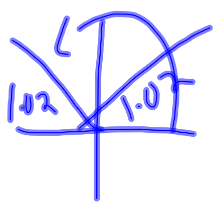
~~$x \approx \sin x$~~



$x = \pi - 0.146$

$x = 2.99$ ✓

$\sin x = 0.853$



$x = \sin^{-1}(0.853)$ ✓

$x = 1.02$

$x = \pi - 1.02 = 2.12$ ✓

$x = 0.146, 1.02, 2.12, 2.99$

$= 0.146 - 2\pi, 1.02 - 2\pi, 2.12 - 2\pi, 2.99 - 2\pi$

$= -6.137, -5.263, -4.163, -3.29$