

7.5 Solving Linear Trigonometric Equations

Remember: Solving Linear Equations

$$2x + 1 = 0$$

Example 1: You are given the equation $2\cos x + 1 = 0$, $0 \leq x \leq 2\pi$

- a) Determine all the solutions in the specified interval.
- b) Make a sketch that will help you verify your answers.

Example 2: Solve $4(2+\cot x) = 7$, where $0 \leq x \leq 360$, correct to one decimal place.

Now you try $3(\tan x + 1) = 2$

Example 3: The depth of the ocean at a swim buoy can be modelled by the function $d(t)=4+2\sin(\pi t/6)$, where d is the depth of the water in metres and t is the time in hours, if $0 \leq t \leq 24$. Consider a day when $t=0$ represents midnight. Determine when the depth of water is 3m.

At home, you should try Example 3, pg 422

$$d(t) = 4 + 3.5\cos(\pi t/6)$$

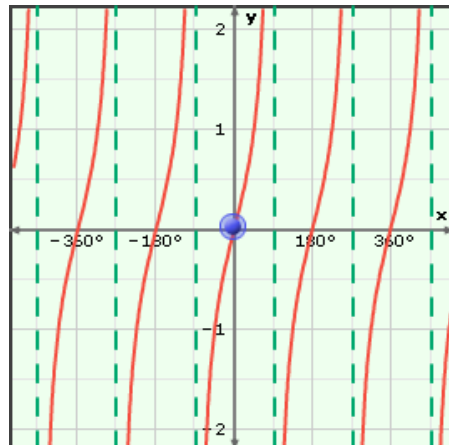
d is depth in metres of water in a cove

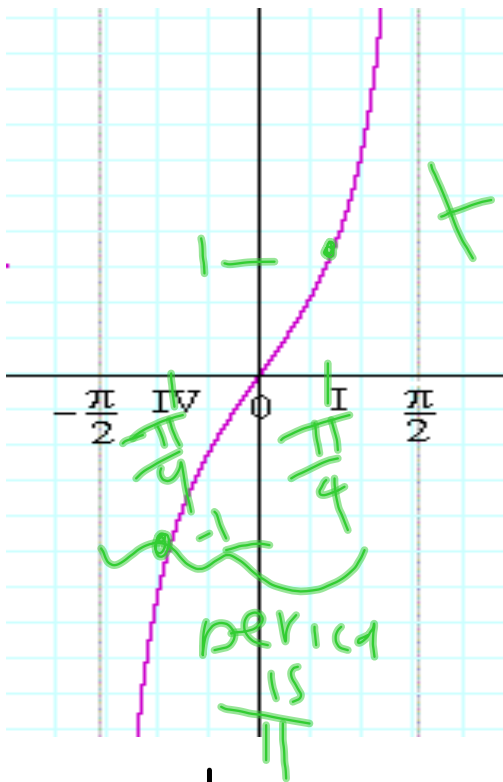
t is time in hours

Jenny can only manoeuvre in water that is at least 2 m deep.

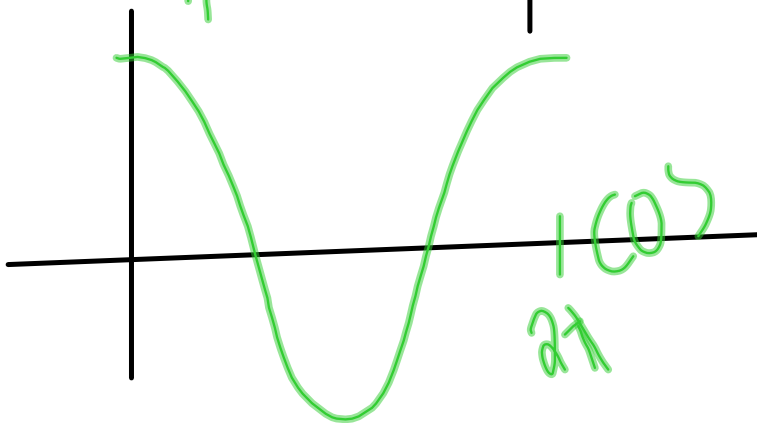
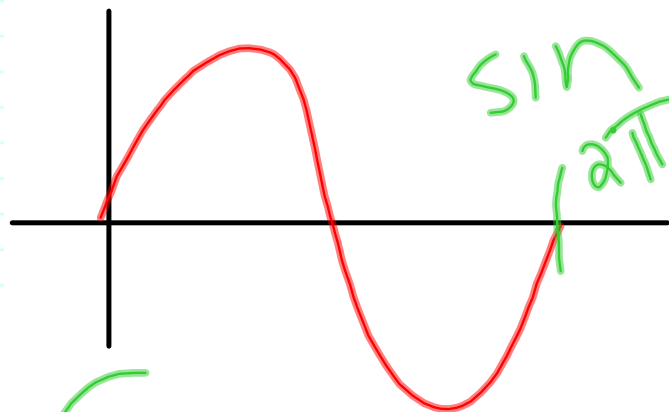
When can she sail?

Example 4: Solve $2\sin x \cos x = \cos 2x$ for the interval $0 \leq x \leq 2\pi$





This is one cycle of the tanx graph

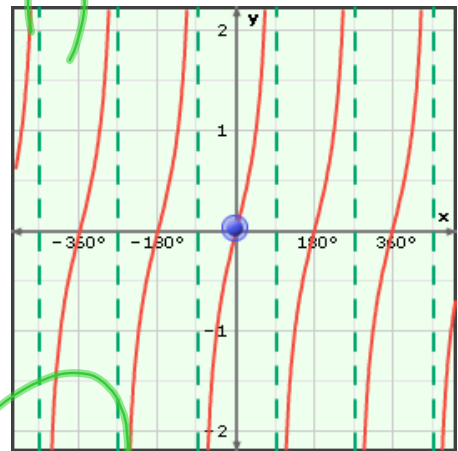


Example 5: Determine the solutions for the equation, where $0 \leq x \leq 3\pi$

$$\tan x = -1$$

$$x = \tan^{-1}(-1)$$

$$x = -\frac{\pi}{4}$$



$$\frac{-\pi}{4},$$

$$\frac{-\pi}{4} + \frac{4\pi}{4} =$$

$$\frac{3\pi}{4}$$

$$\frac{3\pi}{4} + \frac{4\pi}{4} =$$

$$\frac{7\pi}{4}$$

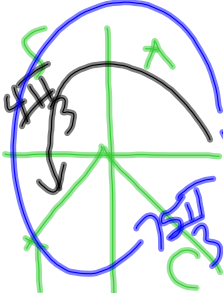
$$= \frac{11\pi}{4}$$

$$\frac{15\pi}{4}$$

Example 6: Determine the solution on the interval $0 \leq x < 2\pi$

$$\sin x = \frac{\sqrt{3}}{2} \quad \sin(3x) = -\frac{\sqrt{3}}{2}$$

Let $\theta = 3x$

$$\sin \theta = -\frac{\sqrt{3}}{2}$$


$$\sin \theta_R = \frac{\sqrt{3}}{2}$$

$$\theta_R = \frac{\pi}{3}$$

$$\theta = \frac{4\pi}{3}, \frac{5\pi}{3} = \frac{4\pi}{3} \times \frac{2}{3}$$

$$3x = \frac{4\pi}{3} \quad 3x = \frac{5\pi}{3}$$

$$x = \frac{4\pi}{9} \quad x = \frac{5\pi}{9}$$

To find next x 's, we must add the period.

$$x = \frac{4\pi}{9} + \frac{6\pi}{9} \quad x = \frac{5\pi}{9} + \frac{6\pi}{9}$$

$$= \frac{10\pi}{9} \quad = \frac{11\pi}{9}$$

$$x = \frac{10\pi}{9} + \frac{6\pi}{9} \quad x = \frac{11\pi}{9} + \frac{6\pi}{9}$$

$$= \frac{16\pi}{9} \quad = \frac{17\pi}{9}$$

$$x = \frac{4\pi}{9}, \frac{5\pi}{9}, \frac{10\pi}{9}, \frac{11\pi}{9}, \frac{16\pi}{9}, \frac{17\pi}{9}$$

$$0 \leq x \leq 2\pi$$

Example: $5\cos x - \sqrt{3} = 3\cos x$

$$5\cos x - 3\cos x = \sqrt{3}$$

$$2\cos x = \sqrt{3}$$

$$\cos x = \frac{\sqrt{3}}{2}$$



$$x = \frac{\pi}{6}$$

$$x = \frac{\pi}{6} \text{ or } \frac{11\pi}{6}$$

$$0 \leq x \leq 2\pi$$

Example: $\cos 4x = -1/\sqrt{2}$

① $4x = \theta$

$$\cos \theta = -\frac{1}{\sqrt{2}}$$

② Find θ_R

$$\cos \theta_R = \frac{1}{\sqrt{2}}$$

③ Find 2 angles

$$\theta_R = \frac{\pi}{4}$$



$$\theta = \frac{3\pi}{4}, \frac{5\pi}{4}$$

④ Change θ to $4x$

$$4x = \frac{3\pi}{4} \quad 4x = \frac{5\pi}{4}$$

$$x = \frac{3\pi}{16} \quad x = \frac{5\pi}{16}$$



⑤ Find other angles by adding the period

$$P = \frac{2\pi}{k}$$

$$P = \frac{2\pi}{4} = \frac{\pi}{2} = \frac{8\pi}{16}$$

$$x = \frac{3\pi}{16} + \frac{8\pi}{16} = \frac{11\pi}{6}$$

$$x = \frac{5\pi}{16} + \frac{8\pi}{16} = \frac{13\pi}{16}$$

$$x = \frac{19\pi}{16}$$

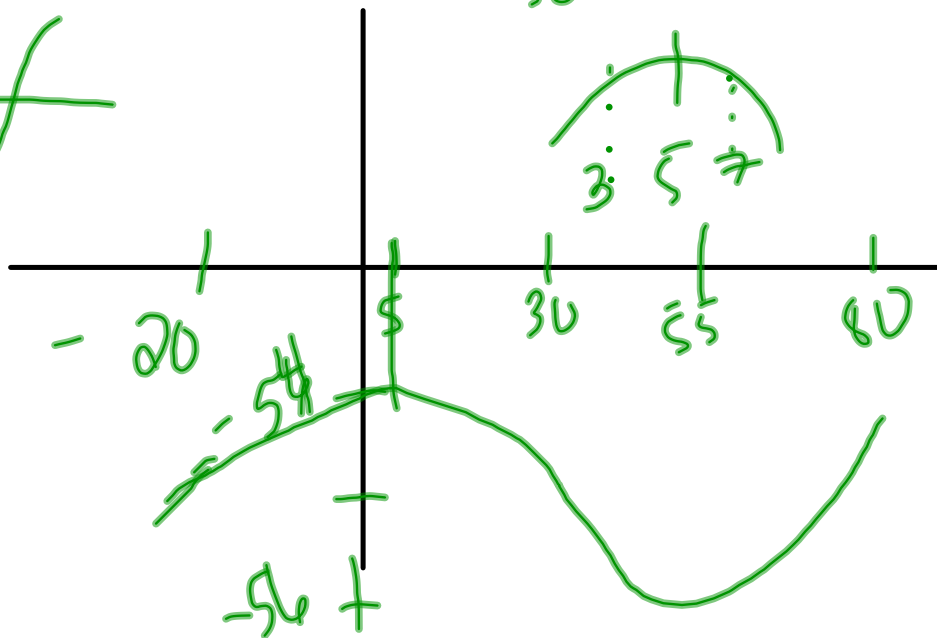
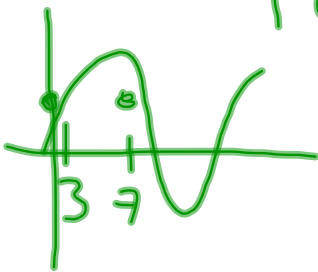
$$= \frac{21\pi}{16}$$

$$x = \frac{27\pi}{16}$$

$$= \frac{29\pi}{16}$$

Example: Explain why the value of the function $f(x) = \sin\left(\frac{\pi}{50}(x+20)\right) - 55$ at $x=3$ is the same as at $x=7$

$$f(x) = \sin\frac{\pi}{50}(x+20) - 55$$



$$= 2\pi \times \frac{20}{\pi}$$

$$= 100$$

2x

3+ Example: $\frac{-5\cot x}{2} + \frac{7}{3} = \frac{-1}{6}$

6=30

$$18 \cancel{36} \left(\frac{-5\cot x}{\cancel{2}} \right) + \overset{12}{\cancel{36}} \left(\frac{7}{\cancel{3}} \right) = \overset{6}{\cancel{36}} \left(\frac{-1}{\cancel{6}} \right)$$

$$-90\cot x + 84 = -6$$

$$-90\cot x = -90$$

$$\cot x = \frac{-90}{-90}$$

$$\cot x = 1$$

$$\tan x = 1$$

$$x = \frac{\pi}{4}$$

$$0 \leq x \leq 4\pi$$

You try: $\frac{2}{\sin x} + 10 = 6$

$$\frac{24\pi}{6}$$

$$\frac{2}{\sin x}$$

$$= 6 - 10$$

$$\frac{2}{\sin x} = -4$$

$$2 = -4 \sin x$$

$$= \sin x$$

$$-4 = \sin x$$

$$= \sin x R$$

$$\frac{11\pi}{6} = x R$$



$$\theta = \frac{7\pi}{6}$$

$$\theta = \frac{11\pi}{6}$$

$$\theta = \frac{11\pi}{6} + \frac{12\pi}{6} = \frac{19\pi}{6}$$

$$\theta = \frac{11\pi}{6} + \frac{12\pi}{6} = \frac{23\pi}{6}$$

$$0 \leq x < 2\pi$$

Example: $2 + 10\sec x - 1 = -18$

$$10\sec x = -18 + 1 - 2$$

$$10\sec x = -19$$

$$\sec x = \frac{-19}{10}$$

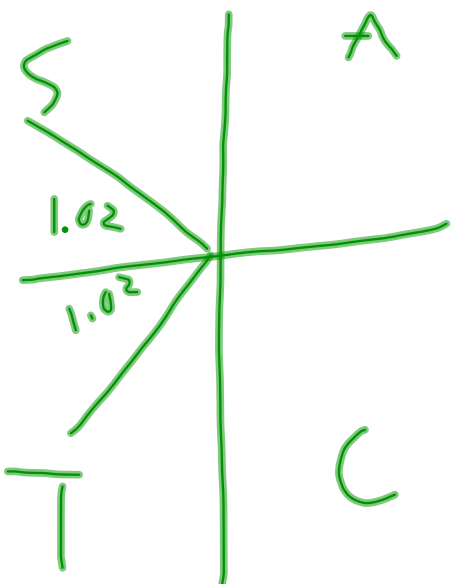
$$\cos x = \frac{-10}{19}$$

$$\cos x_p = \frac{10}{19}$$

$$x_p = \cos^{-1}\left(\frac{10}{19}\right) = 1.02$$

$$x = \pi - 1.02 = 2.12$$

$$x = \pi + 1.02 = 4.16$$



Homework: pg 427- 428

#6,7,8,9,10,11,12,18b