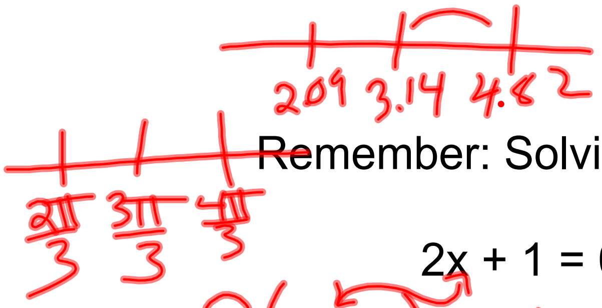
7.5 Solving Linear Trignometric Equations



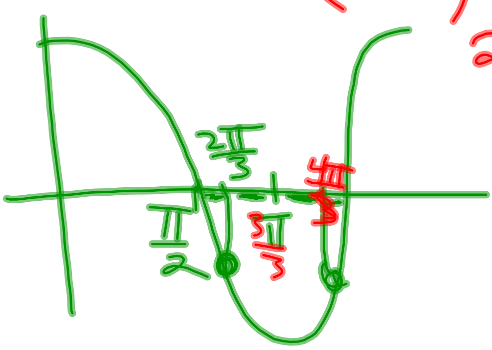
$0 < x < 2\pi$

$2x + 1 = 0$

Remember: Solving Linear Equations

$2x + 1 = 0$

$2(\cos x) + 1 = 0$



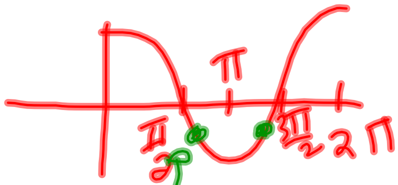
$2\cos x = -1$

$\cos x = \frac{-1}{2}$

$x = \cos^{-1}\left(\frac{-1}{2}\right)$

$= \frac{2\pi}{3}, \; 2.094$

$= \frac{4\pi}{3}, \; 4.82$



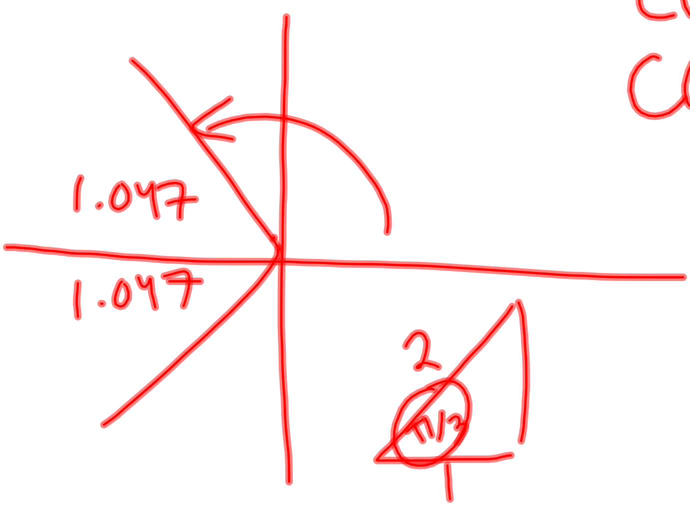
$\cos\theta = -\frac{1}{2}$

$\cos\theta_R = \frac{1}{2}$

$\theta_R = \cos^{-1}\left(\frac{1}{2}\right)$

$= \frac{\pi}{3}$

$= 1.047$



10a) $\cos x \tan^3 x = \sin x \tan^2 x$

$RS = \dfrac{\sin x \tan^2 x}{\tan^2 x}$

$= \sin x$

$LS = \dfrac{\cos x \tan x}{\tan^3 x}$

$= \cos x \dfrac{\sin x}{\cos x}$

$= \sin x$

$\sin^2\theta + \cos^4\theta = \cos^2\theta + \sin^4\theta$

$LS = \sin^2\theta + (\cos^2\theta)^2$

$= \sin^2\theta + (1-\sin^2\theta)^2$

$= \sin^2\theta + 1 - 2\sin^2\theta + \sin^4\theta$

$= -\sin^2\theta + 1 + \sin^4\theta$

$RS = \cos^2\theta + \sin^4\theta$

$= (1-\sin^2\theta) + \sin^4\theta$

$= LS$

5b)

$$1 - \tan^2\theta = \sec^2\theta$$

$\theta = 9$

$$LS = 1 - (\tan 9)^2 \qquad RS = (\sec 9)^2$$

$$= 0.7954 \qquad = \left(\frac{1}{\cos 9}\right)^2$$

$$= 1.2$$

10

$$\frac{1+\tan x}{1+\cot x} = \frac{1-\tan x}{\cot x - 1}$$

$$LS = \frac{1+\frac{s}{c}}{1+\frac{c}{s}} = \frac{\frac{c}{c}+\frac{s}{c}}{\frac{s}{s}+\frac{c}{s}} = \frac{\frac{c+s}{c}}{\frac{s+c}{s}} = \frac{c+s}{c} \div \frac{s+c}{s} = \frac{c+s}{c} \times \frac{s}{s+c} = \frac{s}{c} = \tan\theta$$

$$RS = \frac{1-\frac{s}{c}}{\frac{c}{s}-1} = \frac{\frac{c}{c}-\frac{s}{c}}{\frac{c}{s}-\frac{s}{s}} = \frac{\frac{c-s}{c}}{\frac{c-s}{s}} = \frac{c-s}{c} \times \frac{s}{c-s} = \frac{s}{c} = \tan\theta$$

Example 1: You are given the equation 2cosx + 1 = 0 , $0 \leq x \leq 2\pi$

a) Determine all the solutions in the specified interval.

b) Make a sketch that will help you verify your answers.

$0 \leq x \leq 2\pi$

Example 2: Solve 4(2+cotx) = 7, where $0 \leq x \leq 360$, correct to one decimal plaace.

$4(2+\cot x) = 7$

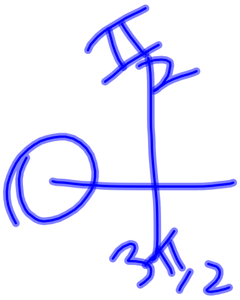
$2+\cot x = \frac{7}{4}$

$2+\cot x = 1.75$

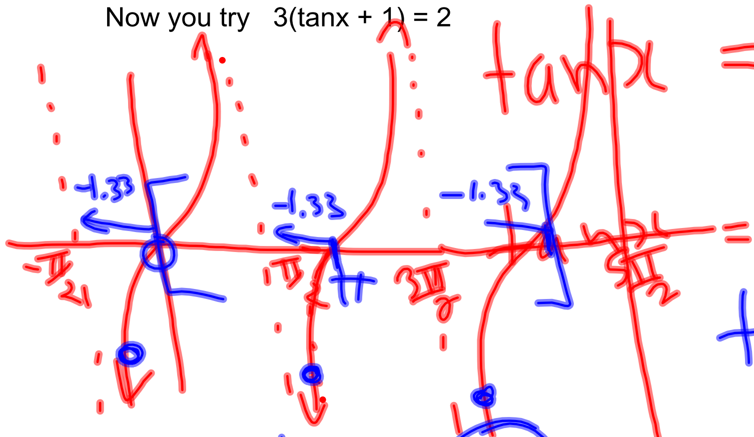
$\cot x = 1.75 - 2$

$\cot x = -0.25$

$\tan x = \frac{1}{-0.25} = -4$



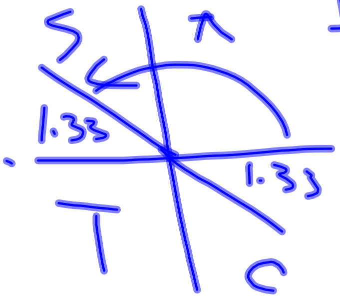
Now you try 3(tanx + 1) = 2



$\tan x = -4$

$x = \tan^{-1}(-4)$

$x = -1.33$



$\tan x = -4$

$\tan x_R = 4$

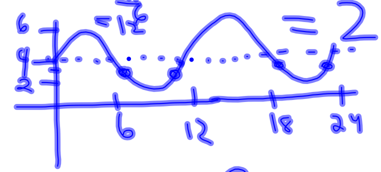
$x_R = \tan^{-1}(4) = 1.33$

$x = \pi - 1.33$ or $x = 2\pi - 1.33$

Example 3: The depth of the ocean at a swim buoy can be modelled by the function $d(t)=4+2\sin(\pi t/6)$, where d is the depth of the water in metres and t is the time in hours, if $0 \le t \le 24$. Consider a day when t=0 represents midnight. Determine when the depth of water is 3m.

$$d(t) = 4 + 2\sin\left(\frac{\pi t}{6}\right)$$

period $= \frac{2\pi}{\frac{\pi}{6}} = 12$

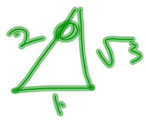


$$= 2\sin\left(\frac{\pi t}{6}\right) + 4$$

$$3 = 2\sin\left(\frac{\pi t}{6}\right) + 4$$

$$3 - 4 = 2\sin\left(\frac{\pi t}{6}\right)$$

$$-1 = 2\sin\left(\frac{\pi t}{6}\right)$$



$$-\frac{1}{2} = \sin\left(\frac{\pi t}{6}\right) \qquad \text{let } x = \frac{\pi t}{6}$$

$$-\frac{1}{2} = \sin(x)$$

$$\sin^{-1}\left(-\frac{1}{2}\right) = x$$

$$-0.52 = x$$

$$3.14 + 0.52 = x$$

or

$$6.28 - 0.52 = x$$

$$3.66 = x$$

or

$$5.76 = x$$

$$3.66 = \frac{\pi t}{6}$$

or

$$5.76 = \frac{\pi t}{6}$$

$$\frac{-1}{2} = \sin x$$

$$\frac{1}{2} = \sin x_R$$

$$\sin^{-1}\left(\frac{1}{2}\right) = x_R$$

$$0.52 = x_R$$

$$x = \pi + 0.52$$

or

$$x = 2\pi - 0.52$$

$$\frac{3.66 \times 6}{\pi} = t$$

$$6.99 = t$$

$$\frac{5.76 \times 6}{\pi} = t$$

$$11.00 = t$$

At home, you should try Example 3, pg 422

$d(t) = 4 + 3.5\cos(\pi t/6)$

d is depth in metres of water in a cove
t is time in hours
Jenny can only manoeuvre in water that is at least 2 m deep.
When can she sail?

Example 4: Solve 2sinxcosx = cos2x for the interval $0 \le x \le 2\pi$

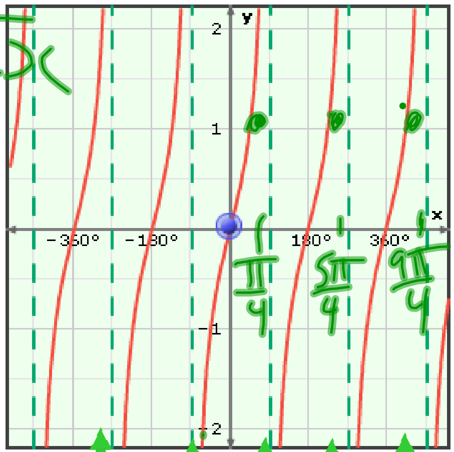
$$2\sin x\cos x = \cos 2x$$

$$\frac{\sin 2x}{\cos 2x} = \frac{\cos 2x}{\cos 2x}$$

$$\frac{\sin\theta}{\cos\theta} \qquad \frac{\sin(2x)}{\cos(2x)} = 1$$

$$\tan\theta \qquad \tan 2x = 1$$

$$\text{let } \theta = 2x \qquad \tan\theta = 1$$



$$\tan\theta = 1$$

$$\theta = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}.$$

$$\frac{2x}{2} = \frac{\frac{9\pi}{4}}{2} \qquad x = \frac{9\pi}{8}$$

$$2x = \frac{\pi}{4} \qquad x = \frac{\pi}{8}$$

$$2x = \frac{5\pi}{4} = \frac{5\pi}{8}$$

* think clearly

Example 4: Solve 2sinxcosx = cos2x for the interval $0 \le x \le 2\pi$

$\frac{16\pi}{8}$

$2\sin x\cos x = \cos 2x$

$\frac{\sin 2x}{\cos 2x} = \frac{\cos 2x}{\cos 2x}$

$\frac{\sin\theta}{\cos\theta}$ $\quad \frac{\sin(2x)}{\cos(2x)} = 1$

$\tan\theta \quad \tan 2x = 1$

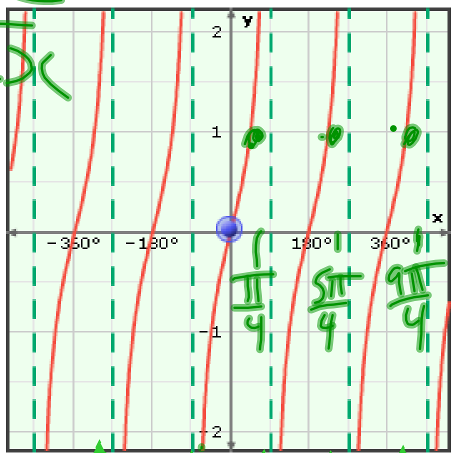
let $\theta = 2x \quad \tan\theta = 1$

$\tan\theta = 1$

$\theta = \tan^{-1}(1)$

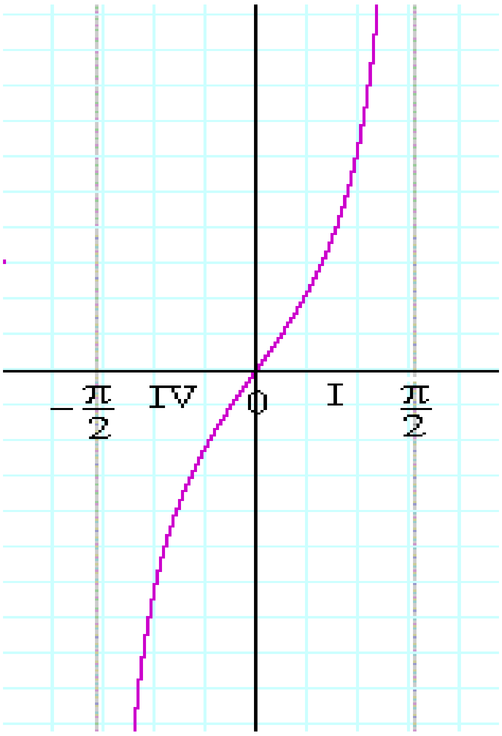
$= 0.785$

$= \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}$



$2x = \frac{\pi}{4}$ or $2x = \frac{5\pi}{4}$

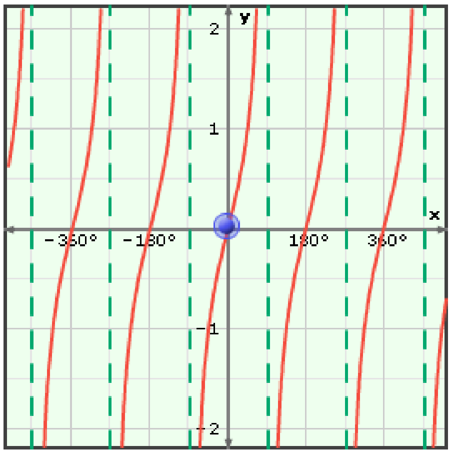
$x = \frac{\pi}{8}$ $\quad x = \frac{5\pi}{8}$



This is one cycle of the tanx graph

Example 5: Determine the solutions for the equation, where $0 \leq x \leq 3\pi$

$\tan x = -1$



Example 6: Determine the solution on the interval $0 \le x \le 2\pi$

$$\sin 3x = -\sqrt{3}/2$$

Example: $5\cos x - \sqrt{3} = 3\text{cox}$

Example: $\cos 4x = -1/\sqrt{2}$

Example: Explain why the value of the function f(x) = sin ((pi/50)(x+20)) - 55
at x= 3 is the same as at x=7

Example: $\frac{-5\cot x}{2} + \frac{7}{3} = \frac{-1}{6}$

You try: $\frac{2}{\sin x} + 10 = 6$

Example: $2 + 10\sec x - 1 = -18$

Homework: pg 427- 428

#6,7,8,9,10,11,12,18b