5.4 Solving Rational Equations

$3x^3 - x^2 + 6x - 7 = 0$

↑ Chap 3

- Factor Theorem
- Q.F

$f(x) = 2x + 4$

$0 = 2x + 4$

$-4 = 2x$

$-2 = x$

$x^2 - 4x + 7 = 0$

Q.F.

or

factor

Example 1: Solve the rational equation.

$$\frac{x-1}{x+5}=0$$

$(x+5)\dfrac{x-1}{x+5}=0(x+5)$

$x-1=0$

$x=1$

$\dfrac{3}{0}\neq 0$

$\dfrac{0}{3}=0$

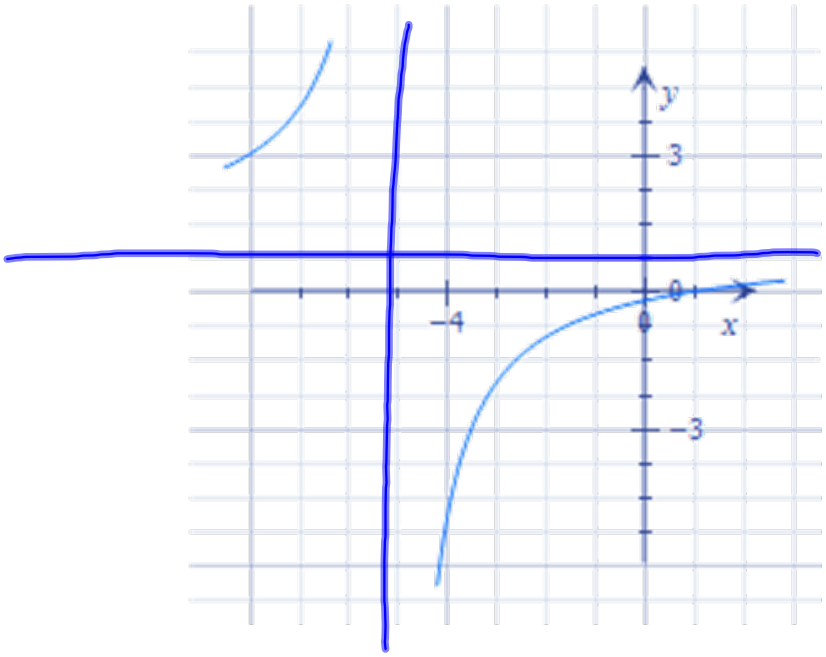
$\dfrac{17}{16}\neq 0$

Example 1: Solve the rational equation.

$$\frac{x-1}{x+5}=0$$

y-intercept $\Rightarrow x=0$

$$f(x)=\frac{x-1}{x+5}$$



V.A.

$x+5=0$

$x=-5$

H.A $\Rightarrow$

$y=$

Example 2: Solve the rational equation.

$$\frac{x+2}{x-5}=\frac{x-1}{x+3}$$

$(x-5)$ $(x+3)$

Restrictions
$x-5\neq 0$
$x\neq 5$
$x\neq -3$

$(x+3)\ x+2=\dfrac{(x-1)(x-5)(x+3)}{x+3}$

$(x+2)(x+2)=(x-1)(x-5)$

$(x+3)(x+2)-(x-1)(x-5)=0$

$x^2+5x+6-(x^2-6x+5)=0$

$x^2+5x+6-x^2+6x-5=0$

$11x+1=0$

$11x=-1$

$x=-\frac{1}{11}$

$2x+7\neq 0$
$2x\neq -7$
$x\neq -\frac{7}{2}$

Example 2: Solve the rational equation.

$$\frac{x+2}{x-5}=\frac{x-1}{x+3}$$

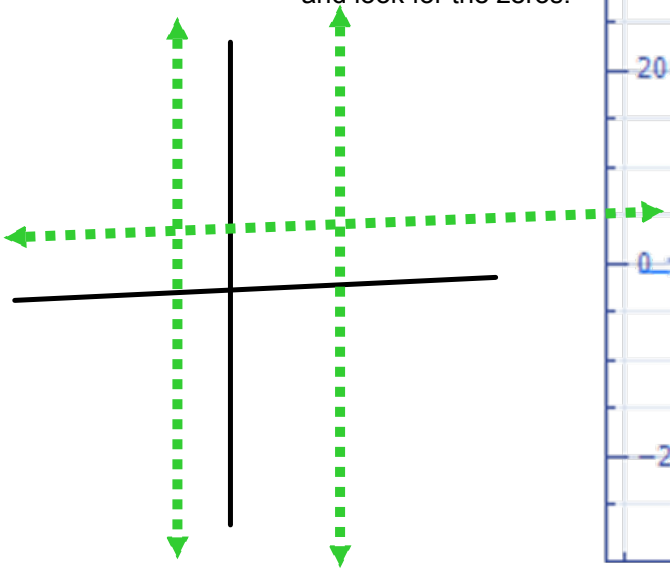
$$\frac{x+2}{x-1}=\frac{x-1}{x+3}$$

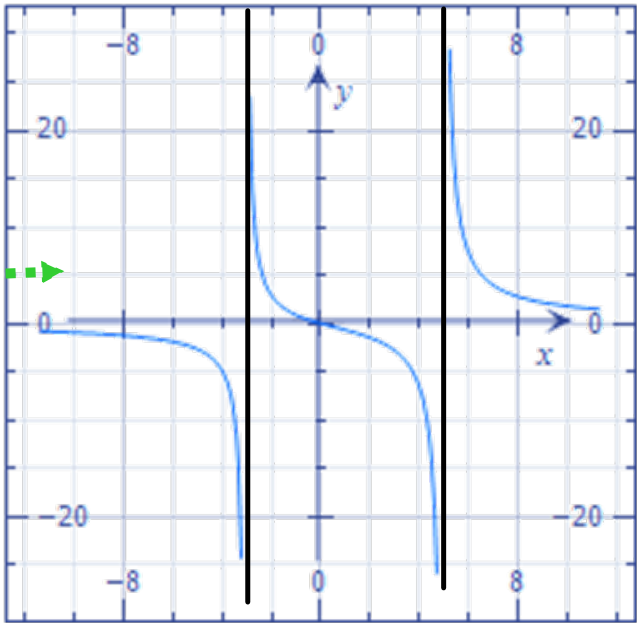
$$\frac{x+2}{x-1}-\frac{x-1}{x+3}=0$$

To check, graph

$$f(x)=\frac{x+2}{x-5}-\frac{x-1}{x+3}$$

and look for the zeros.





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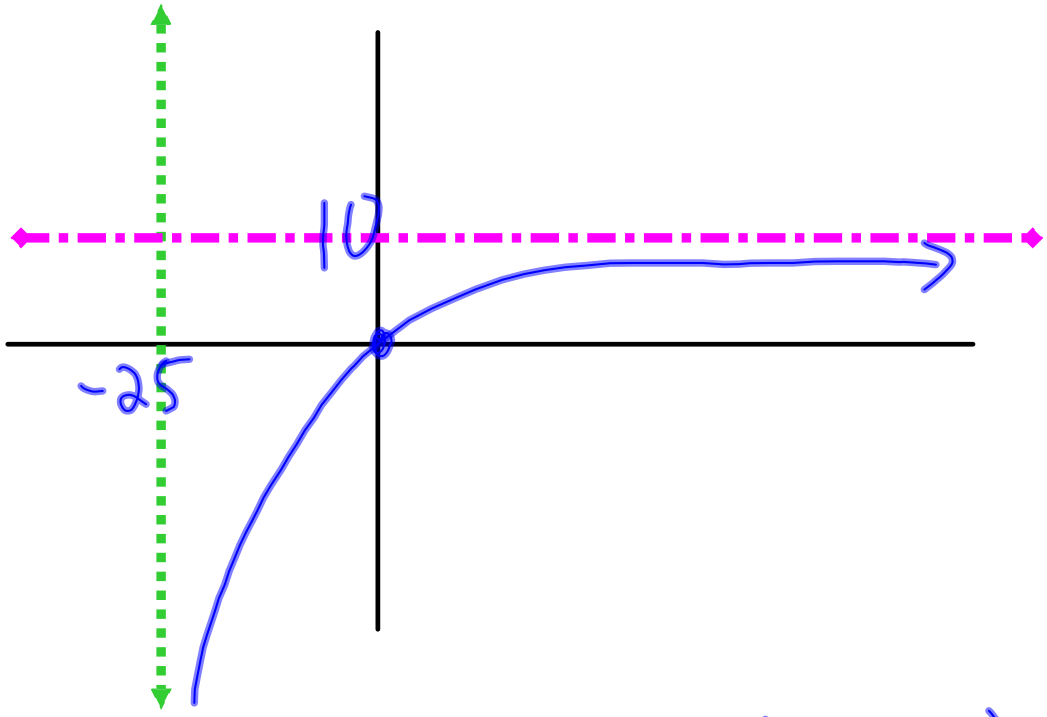
ble 3: Salt water is flowing into a large tank that contains pure water. The concentration of salt, c, in th
t t minutes is given by

$$c(t) = \frac{10t}{25+t} = \frac{10t}{t+25}$$

where c is measured in grams per liter. When does the salt concentration in the tank reach 3.75g/L?



$$(25+t) \times 3.75 = \frac{10t}{25+t} \times (25+t)$$

$$93.75 + 3.75t = 10t$$

$$\frac{93.75}{6.25} = \frac{6.25t}{6.25}$$

$$15 = t$$

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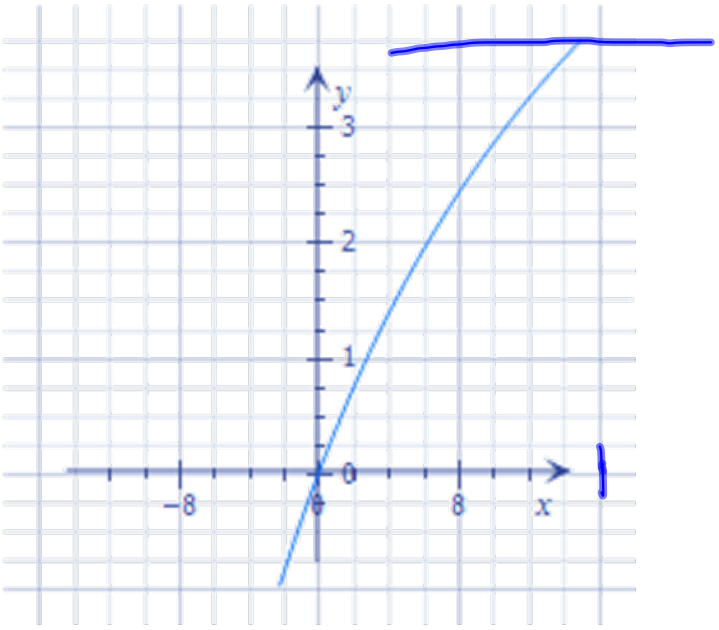
Example 3: Salt water is flowing into a large tank that contains pure water. The concentration of salt, c, in the tank at t minutes is given by

$$c(t) = \frac{10t}{25+t}$$

where c is measured in grams per liter. When does the salt concentration in the tank reach 3.75g/L?



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Example 4: Valary bought a case of concert t-shirts for $500. She kept 3 t-shirts for herself and sold the rest for $620, making a profit of $10 on each t-shirt. How many shirts were in the case?

What should we let the variable x represent?

Suppose 100 t-shirts in the box

How much did each t-shirt cost?

$\frac{\$500}{100} = \$5$/t-shirt

#in the box → 100 #in box − 3

How many does she sell? 97

How much does she sell each t-shirt for?

$\$620/97 = \$6.39$

Profit = 6.39 − 5

= 1.39

$(x-3)\,10 = \frac{620}{x-3} - \frac{500}{x}$

$10(x-3)x = 620x - 500(x-3)$

$10(x-3)x = 620x - 500(x-3)$

$10x^2 - 30x = 620x - 500x + 1500$

$10x^2 - 30x = 120x + 1500$

$10x^2 - 150x = 1500$

$10x^2 - 150x - 1500 = 0$

$10(x^2 - 15x - 150) = 0$

$x = \frac{15 \pm \sqrt{15^2 - 4(-150)}}{2}$

$= 15.84$

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Rita bought a case of t-shirts for $450

She kept 2 and sold rest for $560.

She makes a profit of $10 on each. How many were in the case?

$$10 = \frac{560}{x-2} - \frac{450}{x}$$

$$10(x-2)(x) = 560 \cdot x - 450(x-2)$$

$$10x^2 - 20x = 560x - 450x + 900$$

$$10x^2 - 20x = 110x + 900$$

$$10x^2 - 130x - 900 = 0$$

$$10(x^2 - 13x - 90) = 0$$

$$10(x-18)(x+5) = 0$$

$x = 18$ 18 t-shirts

Homework: pg 286,287 #5,6,7,10,11,12,

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Example 4: Valary bought a case of concert t-shirts for $500. She kept 3 t-shirts for herself and sold the rest for $620, making a profit of $10 on each t-shirt. How many shirts were in the case?

Let x represent the number of t-shirts in the case.

Buying price per t-shirt = 500/x

Selling price per t-shirt = 620/(x-3)

What other number from the question has not yet been used? How can we use it?

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Example 4: Valary bought a case of concert t-shirts for $500. She kept 3 t-shirts for herself and sold the rest for $620, making a profit of $10 on each t-shirt. How many shirts were in the case?

Let x represent the number of t-shirts in the case.

Buying price per t-shirt = 500/x

Selling price per t-shirt = 620/(x-3)

Selling price - Buying price = 10

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Example 5: When they work together, Chris and Jawayed can mow the school field's lawn in 65 minutes. When Jawayed works alone, he can finish the lawn in 5 minutes less time than Chris can when he works alone. When Chris works alone, how long does he take the mow the lawn.

What should we let our first variable represent? Why?

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Example 5: When they work together, Chris and Jawayed can mow the school field's lawn in 65 minutes. When Jawayed works alone, he can finish the lawn in 5 minutes less time than Chris can when he works alone. When Chris works alone, how long does he take the mow the lawn.

Let c represent the time that Chris takes to mow the lawn when he works alone.

Jawayed takes _____ minutes when he works alone.

Compare the rates at which they work. (since we know when you add their rates when they work alone, that should equal the rate when they work together)

Chris's rate - fraction of lawn mowed in one minute

eg. if it takes him 90 minutes to mow the whole lawn, he mows 1/90th per minute

Jawayed's rate - fraction of lawn mowed in one minute

Chris and Jawayed rate when working together.

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