

() () () = 0
4.3 Solving Polynomial Inequalities

$$x^3 - 2x^2 + x - 7 = 0$$

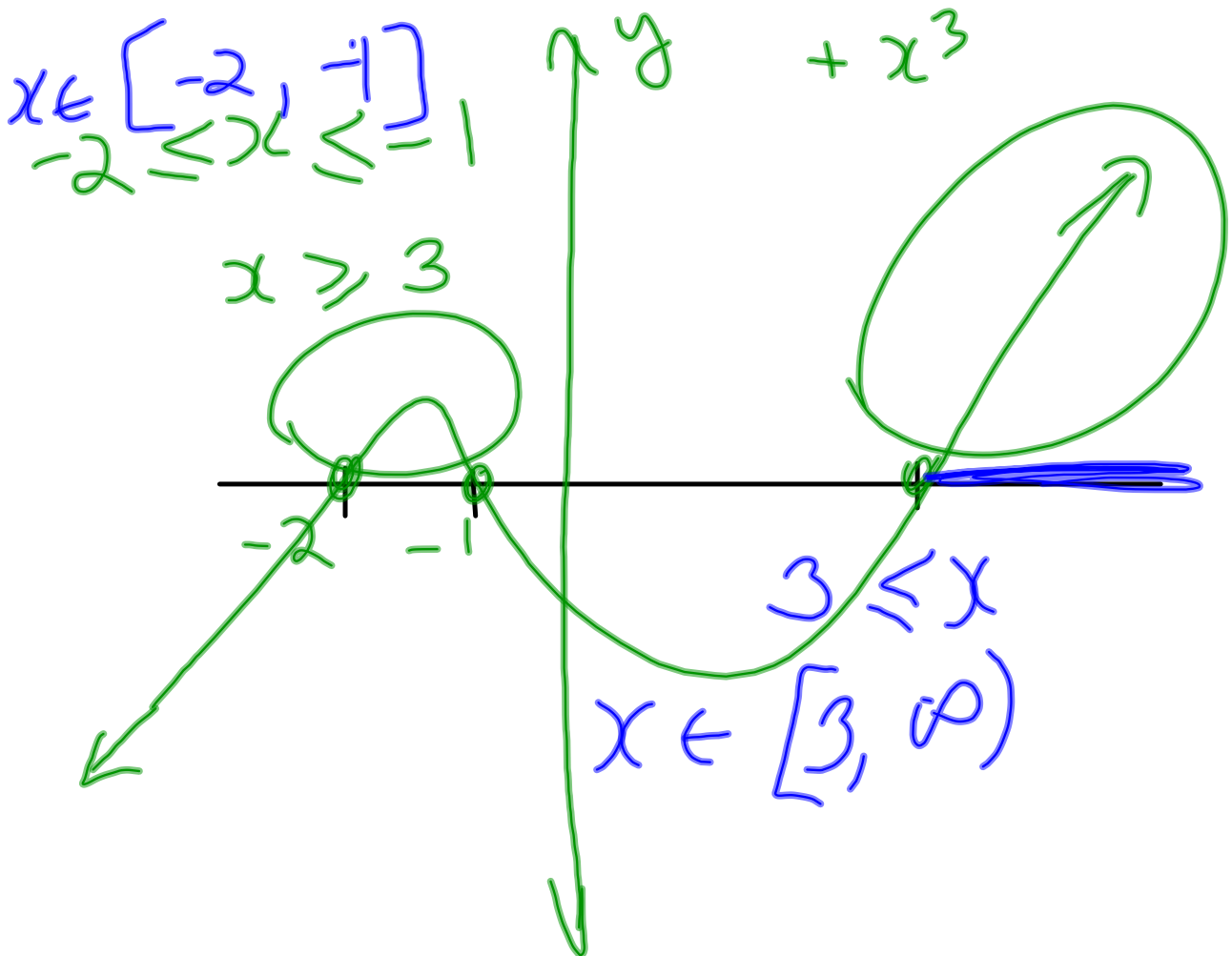
$$2x + 1 = 3$$

$$2x = 2$$

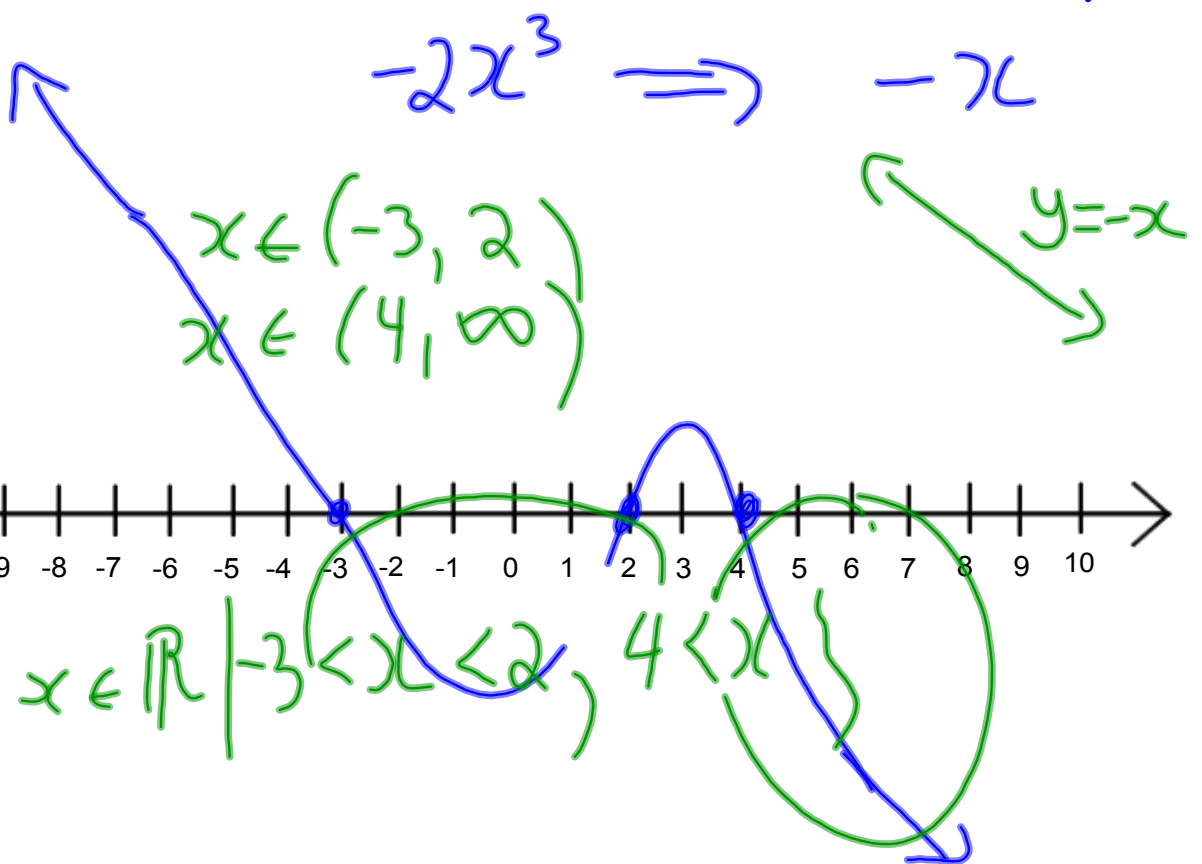
$$x = 1$$

$$2x + 1 > 3$$

Example 1: Solve the inequality $(x+2)(x-3)(x+1) \geq 0$



Example 2: $-2(x-2)(x-4)(x+3) < 0$



Example 3: Solve the inequality

$$x^3 - 2x^2 + 5x + 20 \geq 2x^2 + 14x - 16$$

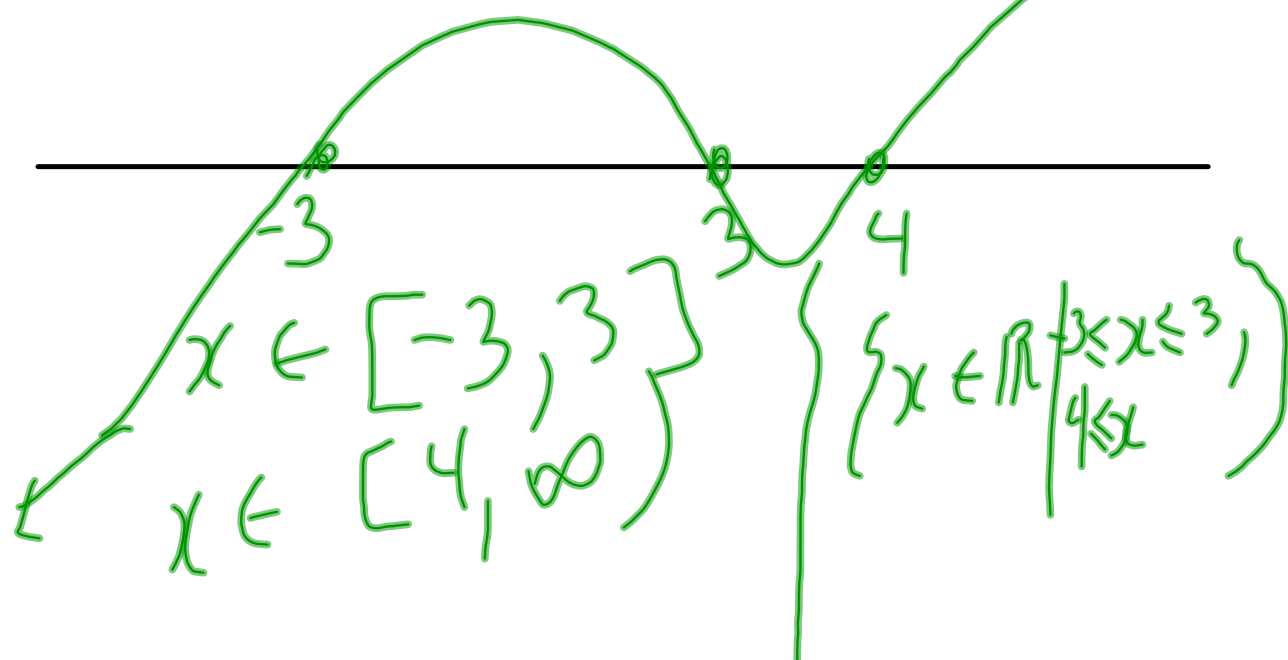
$$x^3 - 2x^2 - 2x^2 + 5x - 14x + 20 + 16 \geq 0$$

$$(x^3 - 4x^2) + 9x + 36 \geq 0$$

$$x^2(x - 4) - 9(x - 4) \geq 0$$

$$(x - 4)(x^2 - 9) \geq 0$$

$$(x - 4)(x + 3)(x - 3) \geq 0$$



Example 4: Solve

$$-2x^4 + 16x^3 - 30x^2 - 16x < -32$$

$$-2x^4 + 16x^3 - 30x^2 - 16x + 32 < 0$$

$$-2(x^4 - 8x^3 + 15x^2 + 8x - 16) < 0$$

$$\begin{array}{r}
 4 < 6 \\
 -2 > -3 \\
 3 < 5 \\
 -9 > -15
 \end{array}
 \begin{array}{c}
 \begin{array}{c} + + + + \\ -3 -2 -1 0 \end{array} \\
 \begin{array}{c} + \\ -15 \quad -9 \end{array}
 \end{array}$$

$$x^4 - 8x^3 + 15x^2 + 8x - 16 > 0$$

Sub in 1 and get 0 $\Rightarrow (x-1)$ is a factor

$$\begin{array}{r}
 x^3 - 7x^2 + 8x + 16 \\
 (x-1) \overline{) x^4 - 8x^3 + 15x^2 + 8x - 16} \\
 \underline{x^4 - x^3} \\
 -7x^3 + 15x^2 \\
 \underline{-7x^3 + 7x^2} \\
 8x^2 + 8x \\
 \underline{8x^2 - 8x} \\
 16x - 16 \\
 \underline{16x - 16} \\
 0
 \end{array}$$

$$x^4 - 8x^3 + 15x^2 + 8x - 16 > 0$$

$$(x-1)(x^3 - 7x^2 + 8x + 16) > 0$$

$(x-4)$ is a factor of $x^3 - 7x^2 + 8x + 16$

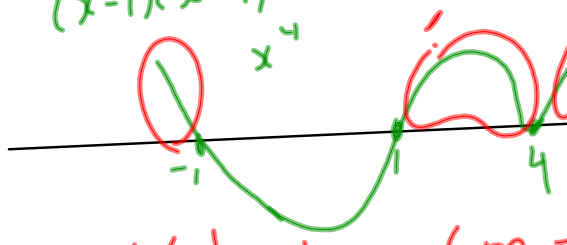
$$\begin{array}{r}
 x^2 - 3x - 4 \\
 (x-4) \overline{) x^3 - 7x^2 + 8x + 16} \\
 \underline{x^3 - 4x^2} \\
 -3x^2 + 8x \\
 \underline{-3x^2 + 12x} \\
 -4x + 16 \\
 \underline{-4x + 16} \\
 0
 \end{array}$$

$$(x-1)(x^3 - 7x^2 + 8x + 16) > 0$$

$$(x-1)(x-4)(x^2 - 3x - 4) > 0$$

$$(x-1)(x-4)(x-4)(x+1) > 0$$

$$(x-1)(x-4)^2(x+1) > 0$$



Example 5: The height in metres of a projectile launched from the top of a building is given by

$$h(t) = -5t^2 + 20t + 15$$

Where t is the time in seconds since it was launched.

- How high was the projectile when it was launched.
- How long is the projectile in the air?
- For what times in the height of the rock greater than 20m ?
- How long is the projectile above a height of 20m?

a)

$$h(0) = -5(0)^2 + 20(0) + 15 = 15$$

b)

$$h(t) = -5(t^2 - 4t - 3)$$

use Q.F.

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{4 \pm \sqrt{16 - 4(1)(-3)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{28}}{2}$$

$$\begin{array}{l} 28 \\ \swarrow \searrow \\ 2 \quad 14 \\ \swarrow \searrow \\ 2 \quad 7 \end{array}$$

$$= \frac{4 \pm \sqrt{2 \cdot 2 \cdot 7}}{2}$$

$$= \frac{4 \pm 2\sqrt{7}}{2}$$

$$= 2 \pm \sqrt{7}$$

$$= -5(t - (2 + \sqrt{7}))(t - (2 - \sqrt{7}))$$

$$0 = -5(t^2 - 4t - 3)$$

$$t = 2 \pm \sqrt{7}$$

Q6

c)

$$-5t^2 + 20t + 15 > 20$$

$$-5t^2 + 20t - 5 > 0$$

$$-5(t^2 - 4t + 1) > 0$$

$$t^2 - 4t + 1 < 0$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\begin{array}{l} 12 \\ \swarrow \searrow \\ 2 \quad 6 \\ \swarrow \searrow \\ 2 \quad 3 \end{array}$$

$$= \frac{4 \pm \sqrt{16 - 4(1)(1)}}{2(1)}$$

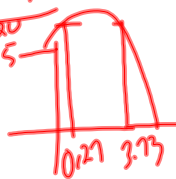
$$= \frac{4 \pm \sqrt{12}}{2} \quad \sqrt{3 \cdot 3} = 3$$

$$= \frac{4 \pm \sqrt{2 \cdot 2 \cdot 3}}{2}$$

$$= \frac{4 \pm 2\sqrt{3}}{2}$$

$$= 2 \pm \sqrt{3}$$

$$= 0.27 \quad 3.73$$



Homework: pg 225 , 226

1cd, 2,5,6,7,8,9,12