3.7 Factoring a Sum or Difference of Cubes

Difference of Squares

$4x^2 - 9$

$(2x + 3)(2x - 3)$

$16y^2 - 25$

$(4y - 5)(4y + 5)$

$a^2 - 100$

$(a + 10)(a - 10)$

$y^3 - 27$

$y^3 - 3^3$

Example 1: Factor the expression x^3-2^3 (x^3-8)

$x^3-8=x^3-2^3$

① x^3-8

Use factor theorem

Sub in $x=2 \Rightarrow 2^3-8=0$

$\therefore (x-2)$ is a factor

② Use long division

$$(x-2)\overline{)x^3+0x^2+0x-8}$$

x^2+2x+4

x^3-2x

$2x^2+0x$

$2x^2-4x$

$4x-8$

$4x-8$

0

③

$\therefore x^3-8=(x-2)(x^2+2x+4)$

$x^3-2^3=(x-2)(x^2+2x+4)$

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Example 2: Factor the expression $(2x)^3 + 3^3$

By the factor theorem we know that f(-3/2)=0 so 2x+3 is a factor

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Example 3: Factor the expression x^3-b^3

If b=3, x^3 -27 =

If b=5, x^3 -125 =

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State a general factorization for the difference of cubes

A^3-B^3

$$x^3 - 8 = x^3 - 2^3$$
$$= (x - 2)(x^2 + 2x + 2^2)$$
$$= \mathbf{(x - 2)(x^2 + 2x + 4)}$$

$$x^3 - 64$$
$$= x^3 - 4^3$$
$$= (x - 4)(x^2 + x4 + 4^2)$$
$$= (x - 4)(x^2 + 4x + 16)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

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State a general factorization for the sum of cubes

$A^3 + B^3$

$$27x^3 + 1 = (3x)^3 + 1^3$$
$$= (3x + 1)((3x)^2 - (3x)(1) + 1^2)$$
$$= \mathbf{(3x + 1)(9x^2 - 3x + 1)}$$

$$8x^3 + 27$$
$$= (2x)^3 + 3^3$$
$$= (2x + 3)[(2x)^2 - (2x)3 + 9]$$
$$= (2x + 3)(4x^2 - 6x + 9)$$

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

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$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

Example 4: Factor $27x^3 + 125$.

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

a b

$(3x)^3 + 5^3 = (3x+5)(9x^2 - 15x + 25)$

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

Example 5: Factor $64x^3-125$

$a^3-b^3=(a-b)(a^2+ab+b^2)$

$64x^3-125 = (4x)^3 - 5^3$

$= (4x-5)(16x^2+20x+25)$

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Example 6: Factor x^9-729

$y = x^9 - 729 = (x^3)^3 - 9^3$

$(x^3)^3 - 9^3 = (x^3 - 9)((x^3)^2 + 9x^3 + 81)$

$= (x^3 - 9)(x^6 + 9x^3 + 81)$

$= (x^3 - 9)(y^2 + 9y + 81)$

let $y = x^3$

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$x^3 - 9 = 0$

$x^3 = 9$

$x = \sqrt[3]{9}$

$= 2.08$

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Example 6: Factor x^9-729

$$y = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-9 \pm \sqrt{81-4(1)(81)}}{2}$$

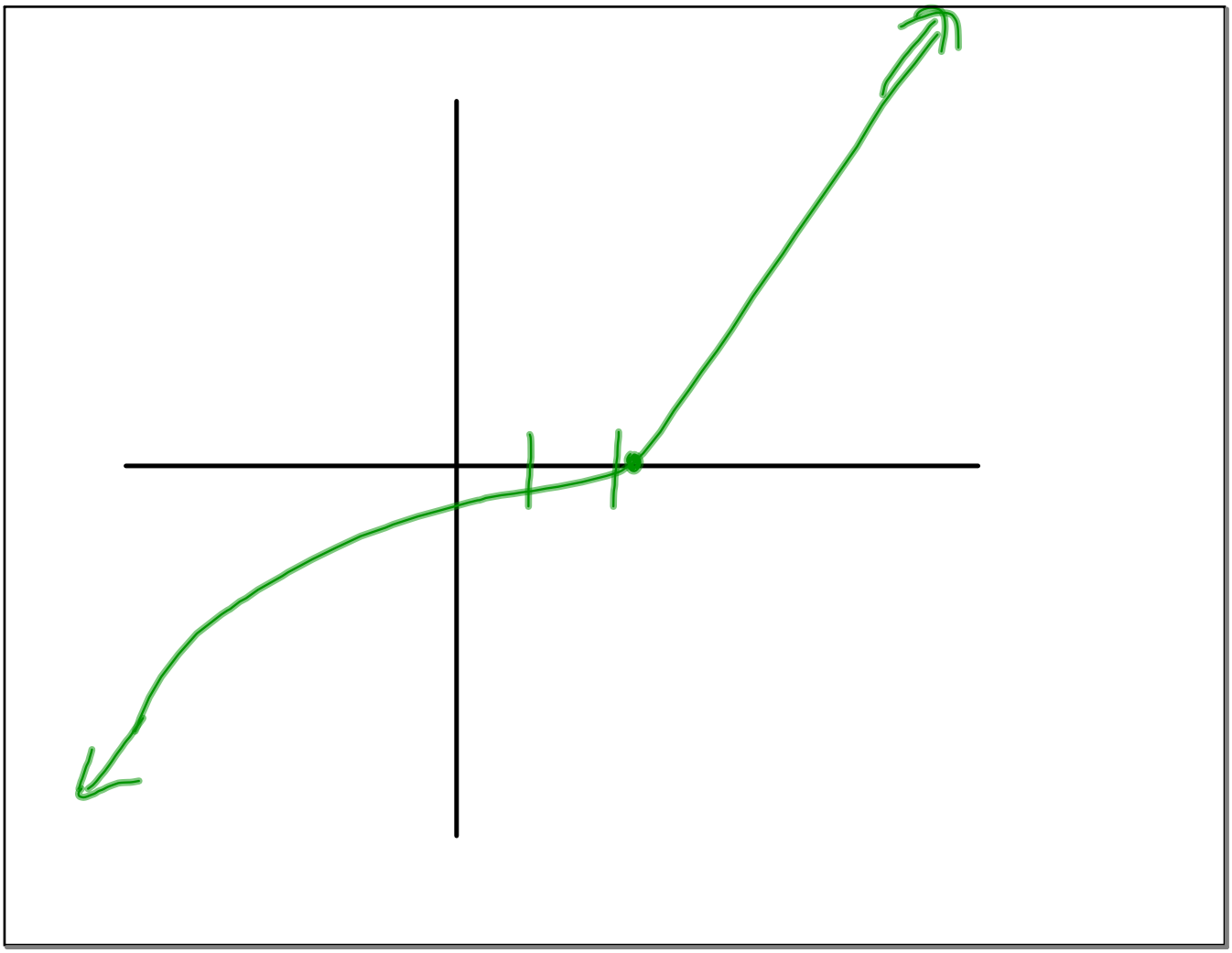
let $y=x^3$ $\Rightarrow$ impossible

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$x^{\frac{1}{3}} = 9$$

$$x = \sqrt[3]{9}$$

$$= 2.08$$



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$a^3+b^3=(a+b)(a^2-ab+b^2)$

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3.

$(3x-7)^3 + (2x+3)^3$

$a \quad b$

$= ((3x-7)+(2x+3))\left((3x-7)^2-(3x-7)(2x+3)+(2x+3)^2\right)$

$= (3x-4)(x^2-14x+49-(2x^2+3x-14x-21)+(4x^2+6x+6x+9))$

$= (3x-4)(x^2-14x+49-2x^2+11x+21+4x^2+12x+9)$

$= (3x-4)(3x^2+9x+79)$

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$2x^3 - x^2 - 145x - 72$

sub 1

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