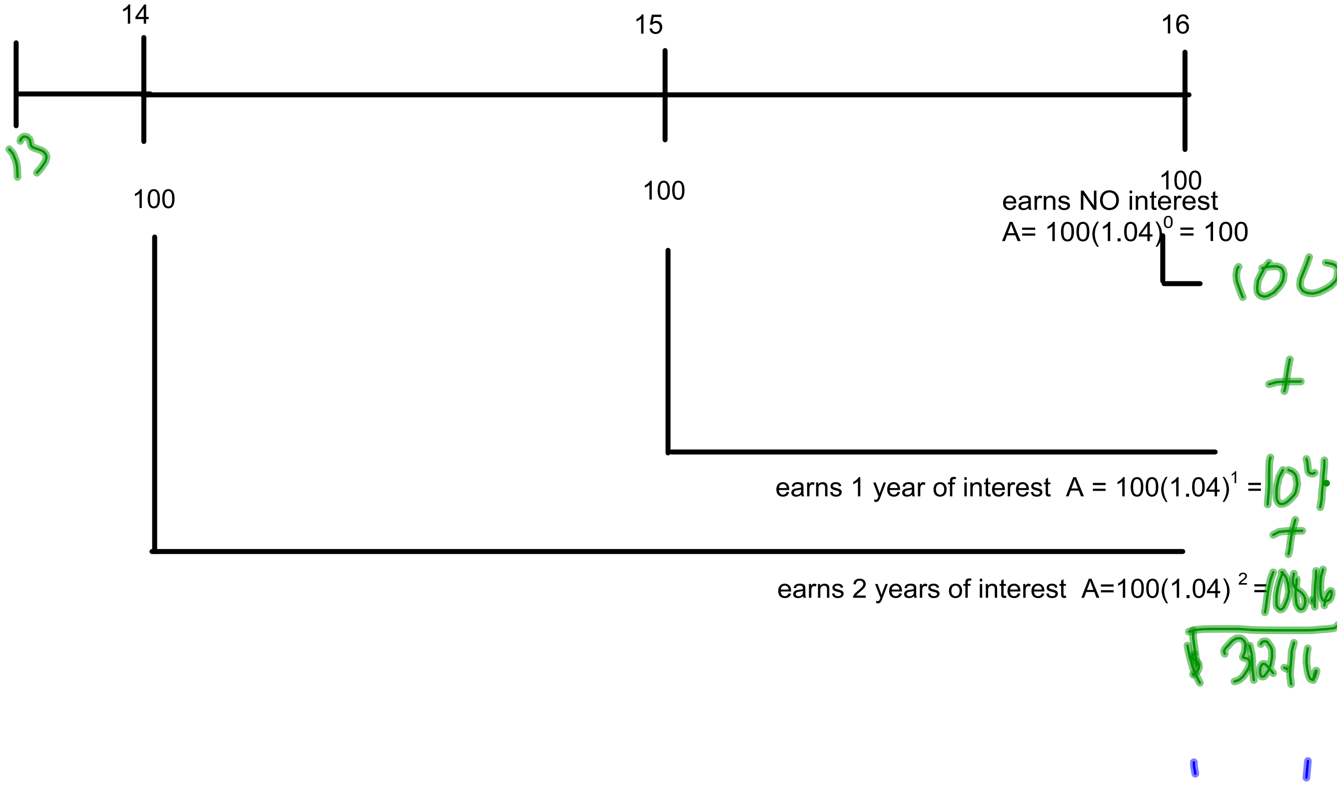
Haidar's mom gives him money for passing each grade. At the end of grade 9, she gave him $100. He opened an account and put the money in the bank. At the end of grade 10, she gave him $100. He put the money in the bank. At the end of grade 11, she gave him $100 and he put it in the bank. How much money does he have in the bank on the day he graduates grade 11 (right after he makes his third deposit) if his bank pays 4% interest/a compounded annually. (Hint: Do all the $100 deposits earn the same amount of interest?)

WHAT IF HIS MOTHER GAVE HIM MONEY FOR ALL HIS BIRTHDAYS UNTIL HE TURNED 40.
WOULD THE FUTURE VALUE BE EASY TO CALCULATE THIS WAY?

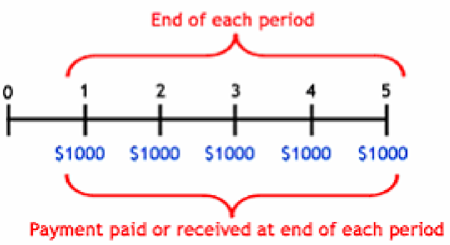
$$S_n = \frac{a(r^n - 1)}{r - 1}$$



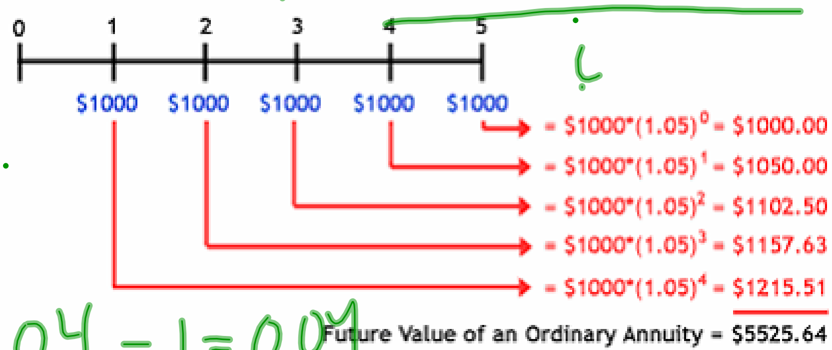
Future Value and Present Value of an Annuity

The Future Value of an Annuity

Example 1: Ahsan plans to deposit $1000 at the end of each year for 5 years. In order to calculate the future value of the annuity, we have to calculate the future value of each cash flow. Let's assume that you are receiving $1,000 every year for the next five years, and you invested each payment at 5%/year compounded annually. The following diagram shows how much you would have at the end of the five-year period:



$$A = \frac{R[(1+i)^n - 1]}{i}$$



$1.04 - 1 = 0.04$

↑ r

$$FV\ of\ Annuity = P\left[\frac{(1+r)^n - 1}{r}\right]$$

$P = Periodic\ Payment$
$r = rate\ per\ period$
$n = number\ of\ periods$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$FV = R\,\frac{[(1 + i)^n - 1]}{i}$$

$$= 100 \cdot \frac{[(1.04)^3 - 1]}{0.04}$$

1.04 [^] [3] [−] [1] [=]

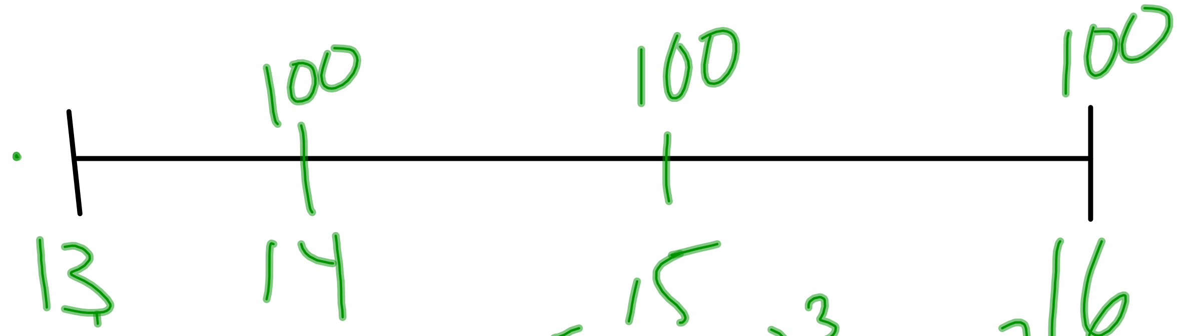
[÷] [0.04] [×] [100]

312.16

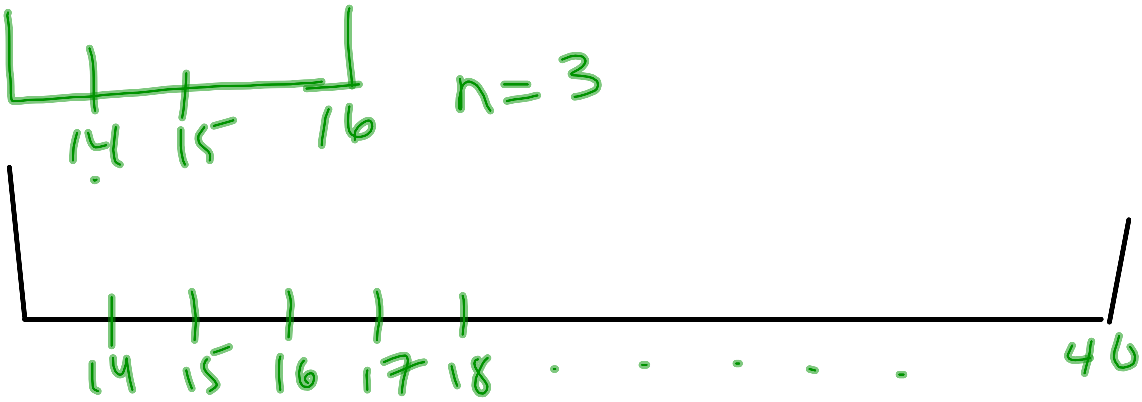
13 14 15 16 17

b) 13–40

$$A = \frac{R\left[(1+i)^n - 1\right]}{i}$$



$$A = \frac{100\left[(1.04)^3 - 1\right]}{0.04} = \$312.16$$



$n = 3$

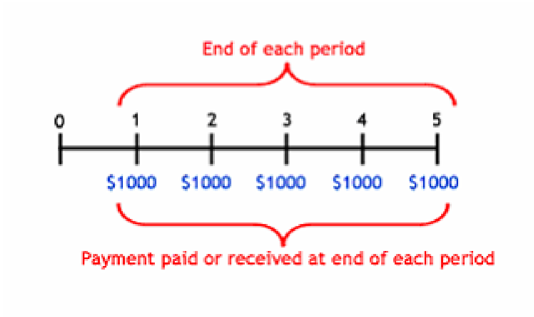
$40 - 14 = 26 + 1$

$n = 27$

$$A = \frac{R\left[(1+i)^n - 1\right]}{i}$$

$$= \frac{100\left((1.04)^{27} - 1\right)}{0.04}$$

$$= \$4708.42$$



$$FV\ of\ Annuity = P\left[\frac{(1+r)^n - 1}{r}\right]$$

$P = Periodic\ Payment$
$r = rate\ per\ period$
$n = number\ of\ periods$ — number of payments

Example 2: $450 is deposited at the end of each quarter for 1.5 years in an account paying 10% per year compounded quarterly.

a) What is the amount of the annuity?

b) How much interest does the annuity earn?

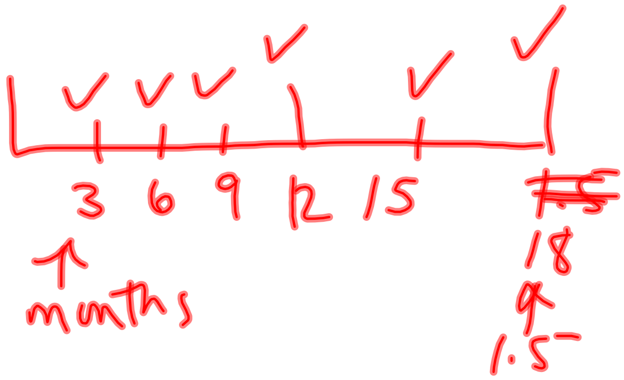
$$A = \frac{R[(1+i)^n - 1]}{i}$$

(2) $i = \frac{0.10}{4}$

$= 0.025$

$n = 4 \times 1.5$

$= 6$



$$A = \frac{R[(1+i)^n - 1]}{i}$$

$= \$2874.48$

$I = A -$ total # of paymnt

$= 2874.48 - (450 \times 6)$

$= 2874.48 - 2700$

$= \$174.48$

Interest $= 2874.48 - 2700$

$= 174.48$

Example 2b: Mawleed deposits $250 at the end of each month for 2 years.
His account pays 6% interest per year, compounded MONTHLY.
a) Determine how much he will have at the end of 2 years.

b) How much interest has he earned.

Example 3: Carmello and Odner are friends. They save for retirement as follows.

- Starting at age 25, Carmello deposits $1000 at the end of each year for 40 years.
- Starting at age 45, Odner deposits $2000 at the end of each year for 20 years.

Suppose each annuity earns 8% per year compounded annually. Who will have the greater amount at retirement?

C	O
$A = \frac{R[(1+i)^n - 1]}{i}$	$A = \frac{R[(1+i)^n - 1]}{i}$
$i = 0.08$	$i = 0.08$
$n = 40$	$n = 20$
$259 056.52	$91 523.92

quarterly

$$A = \frac{R[(1+i)^n - 1]}{i}$$

$i = \frac{0.036}{4} = 0.009$

$n = 4 \times 25 = 100$

$$125\,000 = \frac{R[(1+i)^n - 1]}{i}$$

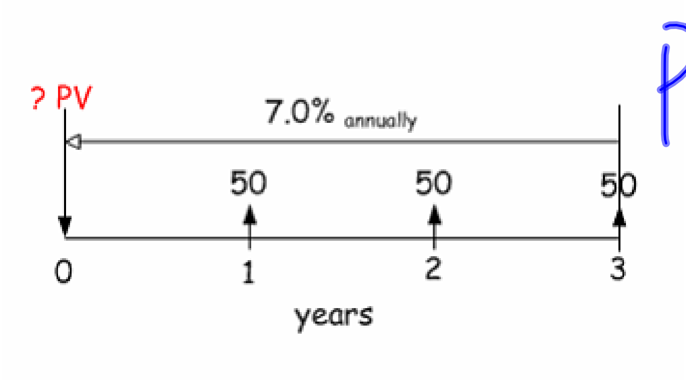
$$125\,000 = \frac{R[(1.009)^{100} - 1]}{0.009}$$

$$\frac{125\,000 \times 0.009}{(1.009^{100} - 1)} = \frac{R[(1.009^{100} - 1)]}{(1.009^{100} - 1)}$$

$776.01 = R

Every quarter (every 3 months) you must deposit $776.01

The Present Value of an Annuity



$$PV = \frac{R\left[1-(1+i)^{-n}\right]}{i}$$

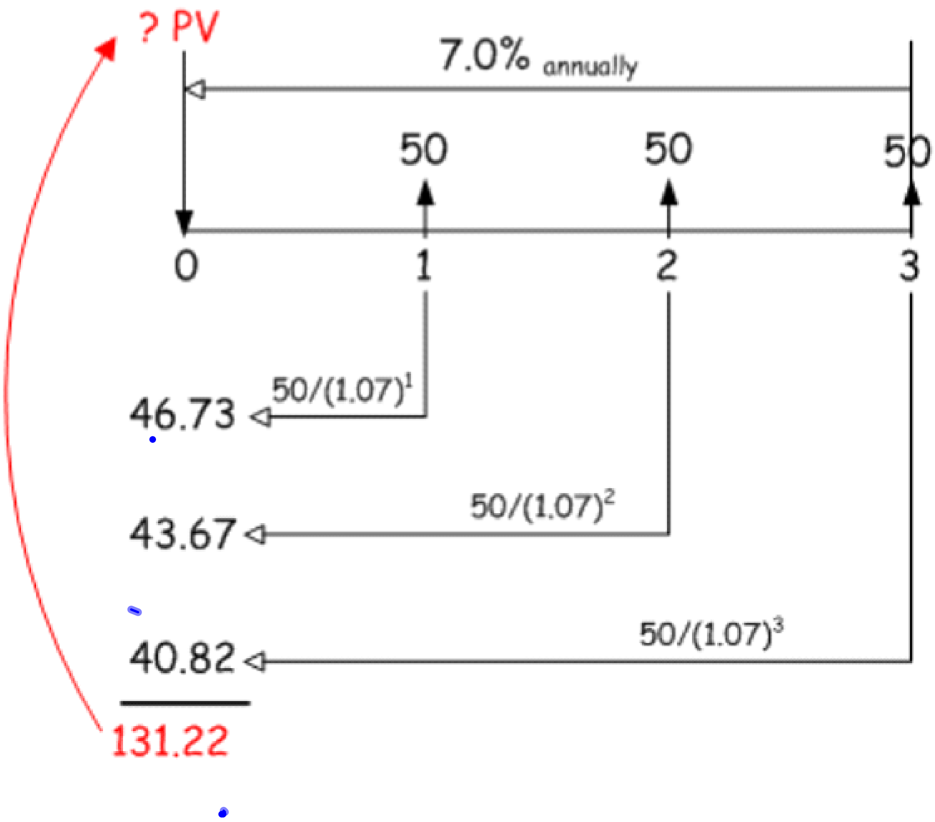
$$= \frac{50\left[1-(1.07)^{-3}\right]}{0.07}$$

Mrs. Thangaraj has just had baby. She wants to put away enough money today so that she can withdraw money to give her son a $50 gift for each of his next three birthdays. How much does she need to deposit if the interest rate is 7.0% annually.

PV

You have a coin that you wish to sell. A potential buyer offers to purchase the coin from you in exchange for a series of three annual payments of $50 starting one year from today. Another buyer is willing to give you $135 dollars today. PV
If the prevailing interest rate is 7% compounded annually, which offer should you take?

= $131.22


? PV
7.0% annually
50
50
50
0
1
2
3
50/(1.07)1
46.73
50/(1.07)2
43.67
50/(1.07)3
40.82
131.22

Regular ~~pay~~ withdrawals

50

7% compounded annually

3 years

$$P\left[\frac{1-(1+r)^{-n}}{r}\right]$$

$P = Periodic\ Payment$
$r = rate\ per\ period$
$n = number\ of\ periods$

$$PV = \frac{R[1-(1+i)^{-n})}{i}$$

$$PV = \frac{50(1-(1.07)^{-3})}{0.07}$$

$$P\left[\frac{1-(1+r)^{-n}}{r}\right]$$

$$P = Periodic\ Payment$$
$$r = rate\ per\ period$$
$$n = number\ of\ periods$$

Example 4: Suzette wants to withdraw $600 at the end of each month for 8 months. Her bank account earns 4.9%/a compounded monthly. How much must Suzette deposit in the bank account today to pay for the withdrawals?

Example 5: Nidaa plans to retire at age 65. She would like to have enough money saved so that she can withdraw $5000 every three months for 20 years. How much must Nidaa deposit at retired at 6% compounded quarterly to provide for this annuity?

PV

65 — 2000 — 2000 — 2000 — 85

$i = \frac{0.06}{4}$

$= 0.015$

$n = 4 \times 20$

$= 80$

deposit a lot of $50

She can make $2000 withdrawals

$PV = \frac{R[1-(1+i)^{-n}]}{i}$

$= \frac{5000[1-(1.015)^{-80}]}{0.015}$

$= \$232\,036.62$

She would need to deposit $232 036.62 into the bank at age 65 to make these withdrawals.

Total withdrawn = $5000 \times 4 \times 20$

$= 400\,000$

Interest earned = Total withdrawn − PV

$= 400\,000 - 232\,036.62$

$= 167\,963.38$

Mr. Walker wants to retire at age 60 and he wants to be able to make monthly withdrawals of $1800 for 25 years.

a) If the interest rate is 3.8% compounded monthly, how much does he need to deposit into his bank at age 60?

b) How much interest does he earn?

$348 259.10

$i = \frac{0.038}{12}$

$n = 25 \times 12$

$= 300$

347 783.12

keeping 5 → 348 553.52

b) Interest = Bigger − Smaller

$= (1800 \times 300) - 348\,259.10$

$= \$191\,740.90$

$A = P(1+i)^n$

You win a lottery. You have 2 choices.

$10 000 in 10 years or FV
10 yearly payments of $900.

$i = 0.046$
$n = 1 \times 10 = 10$

Which should you choose if the prevailing interest rate is 4.6% compounded annually?

$$FV = \frac{R[(1+i)^n - 1]}{i}$$

$$= \frac{900[(1.046)^{10} - 1]}{0.046}$$

$= \$11110.98$

afford \$250/mnth (R) $i = \frac{0.023}{12}$

2.3% / compounded monthly $n = 48$

for 4 years

a) How much can he spend on a car?

$$PV = \frac{R\left[1-(1+i)^{-n}\right]}{i}$$

$$= \frac{250\left(1-\left(1+\frac{0.023}{12}\right)^{-48}\right)}{\left(\frac{0.023}{12}\right)}$$

= \$11 457.07

↑ price tag

Hilliard's total payments = 250 x 12 x 4

= 12 000

Interest = Big - Smaller

= 12000 - 11457.07

= \$545.93

(8.) $R = 250$

$i = \frac{0.052}{4}$

$n = ??$

$FV = 6500$

$$FV = \frac{R[(1+i)^n - 1]}{i}$$

$$6500 = \frac{250[(1.013)^n - 1]}{0.013}$$

$$6500 \times 0.013 = 250(1.013^n - 1)$$

$$\frac{6500 \times 0.013}{250} = 1.013^n - 1$$

$$\frac{6500 \times 0.013}{250} + 1 = 1.013^n$$

Use trial + error