**8.2 Compound Interest**

**Part A: Investigation**

There are two types of interest...

1. **Simple Interest**

Investment with **4% per year** interest

**Year Interest Amount**

0 $700

1 $28 $728

2 $28 $756

3 $28 $784

4 $28 $712

5 $28 $840

2. **Compound Interest**

Investment with **4% interest per year**,

compounded annually.

**Year Interest Amount**

0 $700.00

1 $28.00 $728.00

2 $29.12 $757.12

3 $30.28 $787.40

4 $31.50 $818.90

5 $32.76 $851.66

1. What is true about the interest you earn each year in the simple interest account?
2. How do you get $29.12 as the interest in year 2 for the compound interest account?
3. How do you get $20.28 as the interest in year 3 for the compound interest account?
4. With a compound interest account, you are earning \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ on the interest you have already earned. With a simple interest account you are only earning interest on the amount that you \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.
5. Let the amounts from the simple interest account to be a sequence. Write out the first 4 terms of the sequence.
6. Is it an arithmetic sequence or a geometric sequence?
7. Let the amounts from the compound interest account to be a sequence. Write out the first 4 terms of the sequence.
8. Is it an arithmetic sequence or a geometric sequence?
9. Which type of interest makes an account grow faster? Why does this make sense considering the type of sequences they are?

**Future Value Formula for Compound Interest**

 is the principal (The amount you deposit at the beginning)

 is the accumulated amount or future amount

is the interest rate **per compounding period** (as a decimal)

 is the number of compounding periods

1. Use the formula above to determine the amount you will have after 5 years in the compound interest account.

**Step 1:** Write down the formula.

**Step 2:** Determine P, i and n.

 P = 700 (since that is what you deposit in the beginning)

 i = 4/100 = 0.04 (to change to a decimal, divide by 100)

n = 5 (since you leave the money in the bank for five years, you get interest 5 times – once at the end of each year).

**Step 3**: Sub into the formula.

 A = 700(1+0.04)5

 A = 700 (1.216652902)

 A = 851.66 (we round to two decimal places since money has 2 decimal places)

**Step 4:** Write a concluding statement.

After 5 years we will have $851.66 in the bank. This is the same answer we had in the table above.

**Now you try:** Suppose you put $850 in a bank that paid 2.3% interest for 10 years. How much would you have after 10 years?

**Step 1:** Write down the formula.

**Step 2:** Determine P, i and n.

 P =

 i =

 n =

**Step 3:** Sub into the formula.

**Step 4:** Write a concluding statement.

**Part B: Different Compounding Periods**

Fill in the chart below using the information on **page 489** in your textbook. You will not be able to do the rest of the assignment until you complete this chart.

|  |  |  |  |
| --- | --- | --- | --- |
| Compounding Period  | Meaning  | Interest Rate,  | Term,  |
| annually  |  |  |  |
| semi-annually |  |  |  |
| quarterly  |  |  |  |
| monthly |  |  |  |
| weekly |  |  |  |
| daily |  |  |  |

**Let’s Investigate:**

a. 12% compounded annually

 **12%**

This means you get 12% once a year at the end of the year.

b. 12% semi-compounded annually

 \_\_\_\_\_\_\_ \_\_\_\_\_\_\_

This mean you get a total of 12% in a year. You get 1/2 of it at six months and the other 1/2 at the end of the year.

c. 12% compounded quarterly

 \_\_\_\_\_ \_\_\_\_\_ \_\_\_\_ \_\_\_\_\_\_

This means you get a total of 12% in a year. You get ¼ of it at 3 months, ¼ of it at six months, ¼ of it at 9 months and ¼ of it at 12 months.

d. 12% compounded monthly

This means you get a total of \_\_\_\_\_\_\_ in a year. You get \_\_\_\_\_\_ of it in month # \_\_\_\_\_, \_\_\_\_\_\_ of it in month # \_\_\_\_\_, \_\_\_\_\_\_ of it in month # \_\_\_\_\_, \_\_\_\_\_\_ of it in month # \_\_\_\_\_, \_\_\_\_\_\_ of it in month # \_\_\_\_\_, \_\_\_\_\_\_ of it in month # \_\_\_\_\_, \_\_\_\_\_\_ of it in month # \_\_\_\_\_, \_\_\_\_\_\_ of it in month # \_\_\_\_\_, \_\_\_\_\_\_ of it in month # \_\_\_\_\_, \_\_\_\_\_\_ of it in month # \_\_\_\_\_, \_\_\_\_\_\_ of it in month # \_\_\_\_\_, and \_\_\_\_\_\_ of it in month # \_\_\_\_\_.

**INTEREST = FUTURE VALUE – PRESENT VALUE**

**Part C: Examples**

**Example 1:** Determine how much money you will have if $2000 is invested for 3 years, at 6% per year, compounded semi-annually. How much interest will you earn?

**Step 1:** Underline the word **after** compounded.

**Step 2:** Draw a time line – I split each year up in two since it is compounded semi-annually

0 1 year 2 year 3 years

 **Step 3:** Determine P, i, n

P = 2000 (the amount that was invested at the beginning)

i = 0.06/2 (divide by 2 since it is compounded semi-annually/twice a year)

 = 0.03 (change 6% to a decimal by dividing by 100)

n = 6 (n is the number of times you get interest – since you get interest 2X a year for 3 years, n=6)

 **Step 4:** Plug P, i, n into the formula to find the future amount



A = 2000(1+0.03)6

 A = 2000 (1.03)6

 A = 2388.10 (we round to two decimal places since money has 2 decimal places)

**Step 5:** Determine the interest.

 INTEREST = FUTURE VALUE – PRESENT VALUE

 = 2388.10 – 2000

 = 388.10 **You will earn $388.10 in interest.**

**Now you try:** Suppose Suzette invests $400 in an account that pays 9%/a compounded monthly. How much money will she have after 2 years? How much interest will she earn

**Step 1:** Underline the word **after** compounded.

 **Step 2:** Draw a time line – split each year up in \_\_\_\_\_\_\_ since it is compounded \_\_\_\_\_\_\_\_\_\_\_\_\_\_

 **Step 3:** Determine P, i, n

P = \_\_\_\_\_\_\_\_\_\_\_ (the amount that was invested at the beginning)

i = (divide by \_\_\_\_\_\_ since it is compounded \_\_\_\_\_\_\_\_\_\_\_\_\_\_)

 = (change 9% to a decimal by dividing by \_\_\_\_)

n = (n is the number of times you get \_\_\_\_\_\_\_\_\_\_\_ – since you get interest \_\_\_\_X a year for \_\_\_\_ years, n=\_\_\_\_\_)

 **Step 4:** Plug P, i, n into the formula to find the future amount



 **Step 5:** Determine the interest.

 INTEREST = FUTURE VALUE – PRESENT VALUE

  **She will earn \_\_\_\_\_\_\_\_\_\_\_\_\_ in interest.**

**Example 2:** On her 20th birthday, Nasra invests $5000 at 6%/a compounded semi-annually. She leaves the money in the bank until she retires at age 60. Jackie also invests $5000 on her 20th birthday and leaves it there until she turns 60. Her account offers interest of 6%/a compounded monthly.

a. How much do they each have at age 60?

b. Which will offer more interest – semi-annual compounding or monthly compounding?

**Step 1:** Underline the word **after** compounded.

 **Step 2:** Determine P, i, n for Nasra and also for Jackie

|  |  |  |
| --- | --- | --- |
|  | Nasra | Jackie |
| P |  |  |
| I |  |  |
| N |  |  |

 **Step 3:** Plug P, i, n into the formula to find the future amount for each girl.



|  |  |
| --- | --- |
|  Nasra | Jackie |
|  |  |

**Step 4:** Determine the interest.

|  |  |
| --- | --- |
|  Nasra | Jackie |
| INTEREST = FUTURE VALUE – PRESENT VALUE | INTEREST = FUTURE VALUE – PRESENT VALUE |

Step 6: Decide if semi-annual or monthly compounding yields more interest.

In general, the more frequent the compounding period, the \_\_\_\_\_\_\_\_\_\_\_\_\_\_ the interest.

**Example 3:** On her 15th birthday, Trudy invests $10 000 at 8%/a compounded monthly. When Lina turns 45, she invests $10000 at 8%/a compounded monthly. If both women leave their investments until they are 65, how much more money will Trudy’s investment be worth?

**Step 1:** Underline the word **after** compounded.

**Step 2:** Determine P, i, n for Trudy and also for Lina

|  |  |  |
| --- | --- | --- |
|  | Trudy | Lina |
| P |  |  |
| I |  |  |
| N |  |  |

**Step 3:** Plug P, i, n into the formula to find the future amount for each girl.



|  |  |
| --- | --- |
|  Trudy | Lina |
|  |  |

**Step 4:** Determine the interest.

|  |  |
| --- | --- |
| Nasra | Jackie |
| INTEREST = FUTURE VALUE – PRESENT VALUE | INTEREST = FUTURE VALUE – PRESENT VALUE |

**Step 5:** State whose investment will be worth more.

Check you answer on page 487

In general, the longer you invest your principal, the more \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ you will earn.

**Homework: pg 490 #4a,d,f;5,6,7,10,11,14,15,16,17**