**4.7 – Applications of Exponential Functions**

$$y = a•b^{x}$$

**This is the general form of an equation in exponential form**

**- the** $a$ **is the y-intercept or initial value**

**- the** $b$ **is the growth rate and can be found by calculating the ratio.**

**Example 1:** Determine the equation of the function displayed in the table below.

|  |  |  |
| --- | --- | --- |
| x | y | Ratio (bottom ÷ top) |
| -1 | 1.5 | 3 ÷ 1.5 = 2 |
| 0 | 3 | 6 ÷ 3 = 2 |
| 1 | 6 | 12 ÷ 6 = 2 |
| 2 | 12 | 24 ÷ 12 = 2 |
| 3 | 24 |  |

Since the ratio = 2, $b=2$. Since the y-intercept is 3, $a=3$. We can tell 3 is the y-intercept since you have a y-intercept when $x = 0$.

 Therefore, the equation is $$y = 3•2^{x}$$

 **Now you try:**

|  |  |  |
| --- | --- | --- |
| X | y | Ratio (bottom ÷ top) |
| -2 | -16 |  |
| -1 | -8 |  |
| 0 | -4 |  |
| 1 | -2 |  |
| 2 | -1 |  |

**Try Another:** A biologist tracks the population of a new species of frog over several years. From the table of values, determine an equation that model’s the frog’s population growth.

|  |  |  |
| --- | --- | --- |
| Year | Population | Ratio (bottom ÷ top) |
| 0 | 400 |  |
| 1 | 480 |  |
| 2 | 576 |  |
| 3 | 691 |  |
| 4 | 829 |  |

**CHECK ANSWER ON PAGE 259**

**Example 2:** Wood Buffalo, Alberta had a population of 35 000 in 1996. Its population has grown at an annual rate of about 8%.

1. How long will it take the population to double?

8% = 0.08

1996 Growth = 0.08 x 35 000

 = 2800

1997 Population = 35 000 + 2 800

 = 37 800

1997 Growth = 0.08 x 37 800

 = 3 024

1998 Population = 37 800 + 3 000

 = 40 824

|  |  |  |
| --- | --- | --- |
| Years after 1996When something INCREASES by a certain percent, the ratio will always be 1 + % written as a decimale.g. If the population increases by 5%,the ratio would be 1+0.05 = 1.05 | Population | RATIO |
| 0 | 35 000 | 37 800÷35 000 =1.08 |
| 1 | 37 800 | 40 824÷37 800 =1.08 |
| 2 | 40 824 | 44 090÷40 824 =1.08 |
| 3 | 44 090 | 1.08 |
| 4 | 47 617 | 1.08 |
| 5 | 51 426 | 1.08 |
| 6 | 55 540 | 1.08 |
| 7 | 59 983 | 1.08 |
| 8 | 64 782 | 1.08 |
| 9 | 69 965 | 1.08 |
| 10 | 75 562 | 1.08 |

According to the table, you would have to wait for 10 years, for the population to double. It does not double after 9 years (close but still less than double – double 35 000 is 70 000)

1. What is the equation that models the population over time?

Since the initial value (y-intercept) is 35 000, $a = 35 000$

Since the ratio is 1.08, $b= 1.08$

$$P(t) = 35 000 (1.08)^{t}$$

1. What will the population be in 2010.

$$2010- 1996 = 14$$

Therefore, 2010 is 14 years after 1996 and $t = 14$; sub $t=14$ into the equation.

$$P\left(14\right)= 35 000 \left(1.08\right)^{14}$$

$$ =35 000 \left(2.937193624\right)$$

$$ =102 802 $$

There will be about 102 802 people in 2010.

**You Try:** The municipality of Ottawa has experienced a large population increase in recent years. It population of 1.2 million in 2005 has grown at an annual rate of about 5%.

1. How long will it take for the population to double at his growth rate?
2. What is the equation that models the population over time?
3. What will the population be in 2025?

**Example 3:** A 500g sample of radioactive material has a *half-life* of 200 days. This means that every 200 days, the amount of radioactive material left in a sample is half of the original amount. The mass of radioactive material, in grams, that remains after $t$ days can be modelled by $M\left(t\right)=500(\frac{1}{2})^{\frac{t}{200}}$.

1. Determine the mass that remains after 4 days.

$$M\left(4\right)=500(\frac{1}{2})^{\frac{4}{200}}$$

$$ =500\left(0.9862327045\right)$$

$$ =493 $$

There are about 493 grams left after 4 days.

1. Determine the mass that remains after 4 years.

t is in days, so we must convert 4 years to days. $4 years = 4 x 365 days = 1460 days$

$$M\left(4\right)=500(\frac{1}{2})^{\frac{1460}{200}}$$

$$ =500\left(0.0063457218\right)$$

$$ =3 $$

There are about 3 grams left after 4 years

1. How long does it take for this 500g sample to decay to 70g?

Sub in 70 for M(t), since it is a mass.

$$70=500\left(\frac{1}{2}\right)^{\frac{t}{200}}$$

$$ \frac{70}{500}=\left(\frac{1}{2}\right)^{\frac{t}{200}} we begin to isolate for t by moving $$

$$ 500 to the other side$$

$$0.14=\left(\frac{1}{2}\right)^{\frac{t}{200}} $$

Now you can only use **trial and error** to find $t$.

$$\left(\frac{1}{2}\right)^{2}=0.25$$

$$\left(\frac{1}{2}\right)^{3}=0.125$$

$$\left(\frac{1}{2}\right)^{2.83}=0.14 $$

$$2.83= \frac{t}{200} $$

$$2.83 ×200= t$$

$$t=566 $$

CHECK: $M\left(566\right)=500\left(\frac{1}{2}\right)^{\frac{566}{200}}$

$$=500\left(0.14063\right) $$

$ =70$

**You Try:**

A 200g sample of radioactive material has a *half-life* of 138 days. This means that every 138 days, the amount of radioactive material left in a sample is half of the original amount. The mass of radioactive material, in grams, that remains after $t$ days can be modelled by $M\left(t\right)=200(\frac{1}{2})^{\frac{t}{138}}$.

1. Determine the mass of that remains after 5 years.
2. How long does it take for this 200g sample to decay to 110g?

**CHECK YOUR ANSWER ON PAGE 256**

**Example 4:** A new car cost $30 000. It loses 12% of its value each year after it is purchased. This is called *depreciation.* Determine the value of the car after 42 months.

When something DECREASES by a certain percent, the ratio will always be

 1 - % written as a decimal

e.g. If the car’s value decreases by 5%,the ratio would be 1- 0.05 = 0.95

The initial value of the car is $30 000, therefore $a = 30 000$.

The value decreases by 18% so $b = 1-0.12 = 0.88$

$V\left(t\right)=30 000×0.88^{t}$, where $t$ is the time in years and $V(t)$ is the value of the car.

Must convert 42 months to years since t is time in years.

$$42 months=\frac{42}{12}years= 3.5$$

$$V\left(t\right)=30 000×0.88^{3.5}$$

$$=19178.32$$

Therefore the car is values at $19 178.32 in 42 months.

**You Try:** A new car costs $24 000. It loses 18% of its value each year after it is purchased. This is called *depreciation.* Determine the value of the car after 30 months.

**CHECK YOUR ANSWER ON PAGE 260**

**HOMEWORK: pg 261 #3,4,6,7,8**