Name: Solutions

MBF3C
Exponential Relations

1.

a) Complete the table of values for the two exponential relations, $y = 3 \times 2^x$ and $y = 3 \times 0.5^x$.

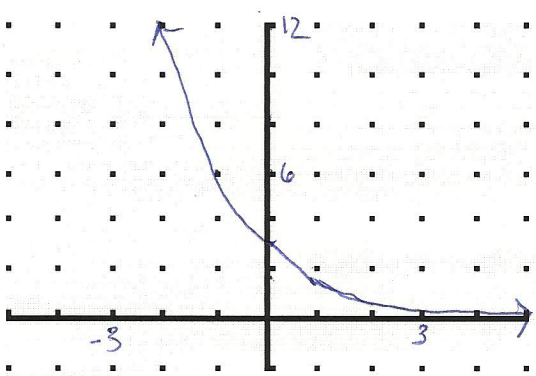
b) Graph both relations in the space provided.

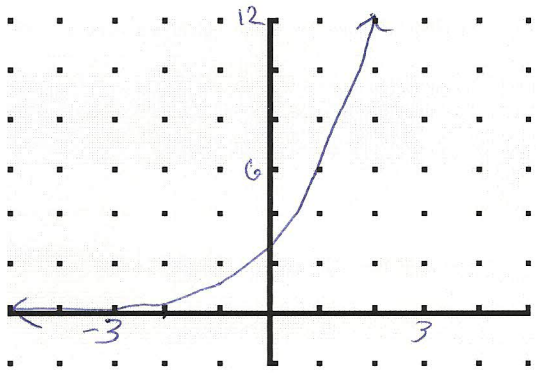
c) How does the base of the exponent affect the graph of the relation?

d) What does the 3 in both equations tell us about the graph of the relation?

X	$y = 3 \times 0.5^x$	Finite Ratio
-2	~~12~~	----------
-1	6	$\frac{1}{2}$ or 0.5
0	3	0.5
1	1.5	0.5
2	0.75	0.5
3	0.375	0.5

X	$y = 3 \times 2^x$	Finite Ratio
-2	0.75	----------
-1	1.5	2
0	3	2
1	6	2
2	12	2
3	24	2





2. Consider the two tables below. Which one describes an exponential relation? Justify your answer. (3 marks)

X	y	Finite Ratio
1	15	
2	45	3
3	135	3
4	405	3
5	1215	3

X	y	Finite Ratio
1	50	
2	200	4
3	450	2.25
4	800	1.8
5	1250	

↑ this table describes an exponential relation b/c the finite ratio is the same.

3. Write as **one power** (where applicable) and then solve. (8 marks)

a) $4^{-8} \times 4^{7} = 4^{-2}$

$= \frac{1}{4^2}$

$= \frac{1}{16}$

b) $(-3)^{-6} \div (-3)^{-4}$

$= (-3)^{-6-(-4)}$

$= (-3)^{-6+4}$

$= (-3)^{-2}$

$= \frac{1}{(-3)^2}$

$= \frac{1}{9}$

c) $(2^{-3})^{-3} = 2^{9}$

$= 512$

4. Simplify and rewrite with a positive exponent.

a) $\frac{1}{3^{-4}} = 3^{4}$

b) $\frac{1}{3^{-4}} \times 3^{-6} = 3^{4} \times 3^{-6}$

$= 3^{4+(-6)}$

$= 3^{4-6}$

$= 3^{-2}$

$= \frac{1}{3^2}$

5. Use exponent rules to simplify each expression.

a) $(3x^{6})(-5x^{-3}) = -15x^{6+(-3)}$

$= -15x^{6-3}$

$= -15x^{3}$

b) $(a^{4}b^{2})^{3} = a^{12}b^{6}$

c) $\left(\frac{12a^{6}b^{4}}{3a^{2}}\right)^{3} = \left(4a^{6-2}b^{4}\right)^{3}$

$= \left(4a^{4}b^{4}\right)^{3}$

$= 4a^{12}b^{12}$

6. Consider the powers of 8^4 and 4^6. Prove that the two powers are the same by rewriting the powers with the same base.

$8^4 = 4096$

$4^6 = 4096$

where $8 = 2^3$

$8^4 = (8)(8)(8)(8)$

$= (2^3)(2^3)(2^3)(2^3)$

$= 2^{3+3+3+3}$

$= 2^{12}$

$4^6 = 4 \times 4 \times 4 \times 4 \times 4 \times 4$

$= 2^2 \times 2^2 \times 2^2 \times 2^2 \times 2^2 \times 2^2$

$= 2^{2+2+2+2+2+2}$

$= 2^{12}$

∴ both 8^4 & 4^6 can be written as 2^{12} and they are equal.

7. Toronto's population, P, is projected to grow until 2050 based on the relation , $P = 2.3(1.08)^n$ where n is the number of years after 2011.

e) Is the population increasing or decreasing? Increasing because 1.08 is greater than "1"

f) By what percent is the population increasing?

8%

g) What was the population in 2011?

2.3 units (million?)

h) What will the population be in 2041?

$P = 2.3(1.08)^{30}$

$\doteq 23.14$ million people

41
−11
30

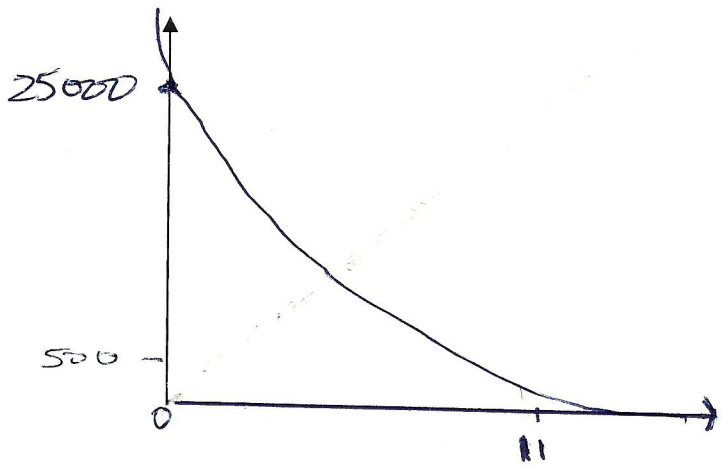
8. The number of bacteria on a wound decrease with each application of an anti-bacterial ointment. The formula, $N = 25000(0.87)^a$ tells you the number of bacteria remaining after a given number of applications.

a) How many bacteria were on the wound at the beginning? 25 000 bacteria

b) By what percent are the number of bacteria decreasing per application?

decreasing by 13%

c) Sketch a graph of this situation.



c) How many bacteria are there after 6 applications?

$N = 25000(0.87)^6$

$N \doteq 10840.66$

∴ after 6 applications, there are approximately 10840.66 bacteria

d) When will then number of bacteria be only 500?

$500 = 25000(0.87)^x$

$\frac{500}{25000} = (0.87)^x$

$0.02 = (0.87)^x$

$0.02 \doteq (0.87)^{10.9}$ } Found by trial & error

∴ $x = 10.9$ so after approximately 11 applications, the bacteria count will be 500

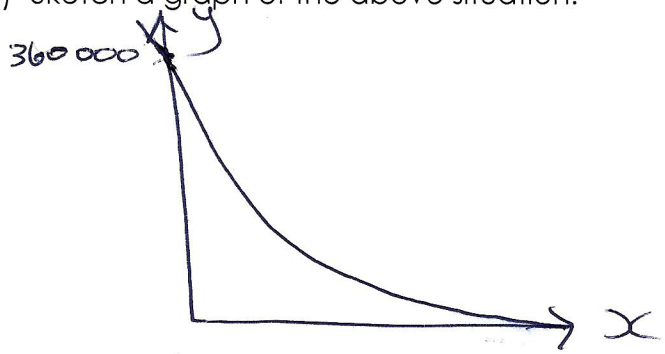
9. Kanata's population, P, is projected to grow for the next 15 years based on the relation $P = 360\,000(1.12)^n$, where n is the number of years after 2011.

a) Is the population increasing or decreasing? Increasing because 1.12 is greater than 1

b) By what percent is it increasing or decreasing?

12%

c) Sketch a graph of the above situation.



c) What was the population in 2011?

360 000 people

d) What will the population be in 2022?

$y = 360000(1.12)^{10}$

$y \doteq 1118105$ people

∴ the population will be

2022
−2011
10 years

e) Can we find the population in 2027?

No, because the relation was only "true" or based on a 15 year projection which would end in 2026. Therefore we can not use the relation to project the population after 2026.

10. Place D'Orleans closes at 6pm on Sunday. At 5:00 pm there are 3100 people in the mall. Each minute, the number of people decrease by 9%.

a) Write an equation for the above situation.

$y = 3100 \times 0.91^x$

$y = a \times (1-i)^n$, where i is 9% or 0.09

$y = 3100 \times 0.91^x$

b) How many people will be in the mall at 5:13pm?

$y = 3100 \times 0.91^{13}$

$y \doteq 910$

∴ approximately 910 will be in the mall.

c) When will there be only 250 people in the mall?

$250 = 3100 \times 0.91^x$

$\frac{250}{3100} = 0.91^x$

$0.0806 = 0.91^x$

$0.0806 = 0.91^{26.5}$

$\therefore x = 26.5$

∴ after 26.5 minutes there will be 250 people in the mall.

d) When will the number of people in the mall be half of the amount there were at 5:00pm?

$3100 \div 2 = 1550$

$y = 3100 \times 0.91^x$

$1550 = 3100 \times 0.91^x$

$\frac{1550}{3100} = 0.91^x$

$0.5 = 0.91^x$

$0.5 \doteq 0.91^{7.52}$

∴ $x \doteq 7.52$ which means at approximately 5:07:52 the mall would have half the amount of people that there were at 5:00.