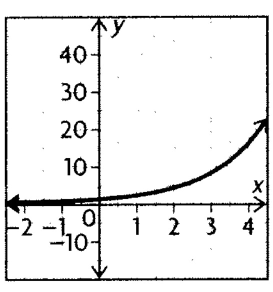
CHAPTER 8
Exponential and Logarithmic Functions

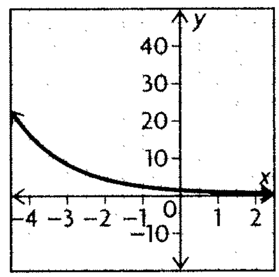
Getting Started, p. 446

1. a) $\frac{1}{5^2} = \frac{1}{25}$
b) 1
c) $\sqrt{36} = 6$
d) $\sqrt[3]{125} = 5$
e) $-\sqrt{121} = -11$
f) $\left(\sqrt[3]{\frac{27}{8}}\right)^2 = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$
2. a) $3^{(2+5)} = 3^7 = 2187$
b) $(-2)^{(12+(-10))} = (-2)^2 = 4$
c) $10^{(9-6)} = 10^3 = 1000$
d) $7^{(6+(-3)-(-1))} = 7^4 = 2401$
e) $8^{(2)\left(\frac{1}{3}\right)} = 8^{\frac{2}{3}} = 4$
f) $4^{\left(\frac{3}{4}+\frac{1}{4}-\frac{1}{2}\right)} = 4^{\frac{1}{2}} = \sqrt{4} = 2$
3. a) $(2m)(2m)(2m) = 8m^3$
b) $a^{-8}b^{-10} = \frac{1}{a^8b^{10}}$
c) $\sqrt{16x^6} = 4|x|^3$
d) $x^{(5-2)}y^{(2-1)} = x^3y$
e) $(-d^4)\left(\frac{c^2}{d^2}\right) = -d^{(4-2)}c^2 = -d^2c^2$
f) $\left(\frac{1}{\sqrt[3]{x^3}}\right)^{-1} = \sqrt[3]{x^3} = x$
4. a)



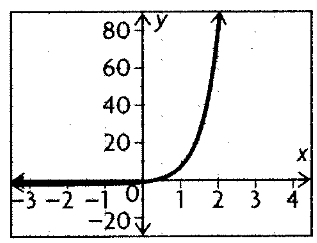
$D = \{x \in \mathbf{R}\}$, $R = \{y \in \mathbf{R} | y > 0\}$,
y-intercept 1; horizontal asymptote $y = 0$

b)



$D = \{x \in \mathbf{R}\}$, $R = \{y \in \mathbf{R} | y > 0\}$,
y-intercept 1, horizontal asymptote $y = 0$

c)



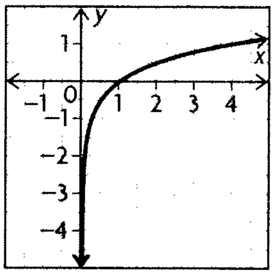
$D = \{x \in \mathbf{R}\}$, $R = \{y \in \mathbf{R} | y > -2\}$,
y-intercept -1, horizontal asymptote $y = -2$
5. a) i) add 6, divide by 3; $y = \frac{x+6}{3}$
ii) add five, take the square root;
$y = \pm\sqrt{x+5}$
iii) divide by 6, take the cube root; $y = \sqrt[3]{\frac{x}{6}}$
iv) subtract 3, take the square root, add 4;
$y = \pm\sqrt{x-3} + 4$
b) The inverses of **(i)** and **(iii)** are functions.
6. a) 12 h ÷ 4 h per doubling = 3
$(100)(2^3) = 800$ bacteria
b) 24 h ÷ 4 h per doubling = 6
$(100)(2^6) = 6400$ bacteria
c) 3.5 days × 24 h/day ÷ 4 h per doubling = 21
$(100)(2^{21}) = 209\,715\,200$
d) 7 days × 24 h/day ÷ 4 h per doubling = 42
$100(2^{42}) = 4.4 \times 10^{15}$
7. x = time in years y = ending population
$y = 15\,000(1 - 0.012)^x$
$y = 15\,000(0.988)^{15}$
$y = 12\,515$ people

8.

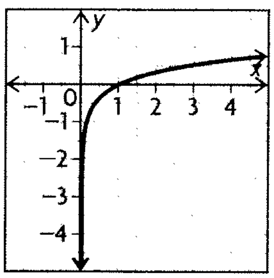
Similarities	Differences
• same y-intercept • same shape • same horizontal asymptote • both are always positive	• one is always increasing, the other is always decreasing • different end behaviour • reflections of each other across the y-axis

8.1 Exploring the Logarithmic Function, p. 451

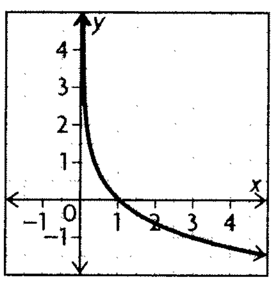
1. a) Inverse function: $x = 4^y$ or $f^{-1}(x) = \log_4 x$



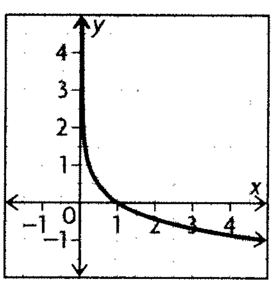
b) Inverse function: $x = 8^y$ or $f^{-1}(x) = \log_8 x$



c) Inverse function: $x = \left(\frac{1}{3}\right)^y$ or $f^{-1}(x) = \log_{\left(\frac{1}{3}\right)} x$



d) Inverse function: $x = \left(\frac{1}{5}\right)^y$ or $f^{-1}(x) = \log_{\left(\frac{1}{5}\right)} x$



2. To write the inverse in exponential form, replace $f(x)$ by y and then switch x and y. Then rewrite in logarithmic form.

a) i) $y = 4^x$

$x = 4^y$

ii) $\log_4 x = y$

b) i) $y = 8^x$

$x = 8^y$

ii) $\log_8 x = y$

c) i) $y = \left(\frac{1}{3}\right)^x$

$x = \left(\frac{1}{3}\right)^y$

ii) $\log_{\frac{1}{3}} x = y$

d) i) $y = \left(\frac{1}{5}\right)^x$

$x = \left(\frac{1}{5}\right)^y$

ii) $\log_{\frac{1}{5}} x = y$

3. All the graphs have the same basic shape, but the last two are reflected over the x-axis, compared with the first two. All the graphs have the same x-intercept, 1. All have the same vertical asymptote, $x = 0$.

4. Locate the point on the graph that has 8 as its x-coordinate. This point is (8, 3). The y-coordinate of this point is the solution to $2^y = 8$, $y = 3$.

5. a) $x = 3^y$ **c)** $x = \left(\frac{1}{4}\right)^y$

b) $x = 10^y$ **d)** $x = m^y$

6. a) $\log_3 x = y$ **c)** $\log_{\frac{1}{4}} x = y$

b) $\log_{10} x = y$ **d)** $\log_m x = y$

7. a) $x = 5^y$ **c)** $x = 3^y$

b) $x = 10^y$ **d)** $x = \left(\frac{1}{4}\right)^y$

8. a) $y = 5^x$ **c)** $y = 3^x$

b) $y = 10^x$ **d)** $y = \left(\frac{1}{4}\right)^x$

9. a) $2^x = 4; x = 2$ **d)** $5^x = 1; x = 0$

b) $3^x = 27; x = 3$ **e)** $2^x = \frac{1}{2}; x = -1$

c) $4^x = 64; x = 4$ **f)** $3^x = \sqrt{3}; x = \frac{1}{2}$

10. If a positive base is raised to any power, the resulting value is positive. There is therefore no way to raise positive 3 to a power and wind up with negative 9.

11. a) $-2 = \log_2 x$
$2^{-2} = x$
$x = \frac{1}{4}$
$\left(\frac{1}{4}, -2\right)$
$-1 = \log_2 x$
$2^{-1} = x$
$x = \frac{1}{2}$
$\left(\frac{1}{2}, -1\right)$
$0 = \log_2 x$
$2^0 = x$
$x = 1$
$(1, 0)$
$1 = \log_2 x$
$2^1 = x$
$x = 2$
$(2, 1)$
$2 = \log_2 x$
$2^2 = x$
$x = 4$
$(4, 2)$

b) $-2 = \log_{10} x$
$10^{-2} = x$
$x = \frac{1}{100}$
$\left(\frac{1}{100}, -2\right)$
$-1 = \log_{10} x$
$10^{-1} = x$
$x = \frac{1}{10}$
$\left(\frac{1}{10}, -1\right)$
$0 = \log_{10} x$
$10^0 = x$
$x = 1$
$(1, 0)$
$1 = \log_{10} x$
$10^1 = x$
$x = 10$
$(10, 1)$
$2 = \log_{10} x$
$10^2 = x$
$x = 100$
$(100, 2)$

8.2 Transformations of Logarithmic Functions, pp. 457–458

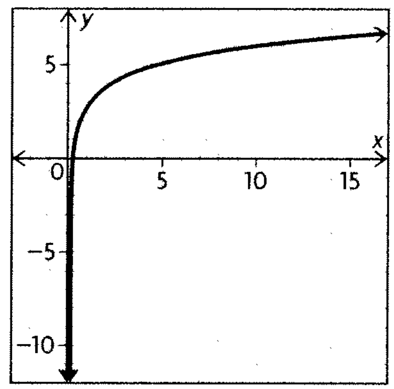
1. The general logarithmic function is
$f(x) = a \log_{10}(k(x - d)) + c$
a) $a = 3$; produces a vertical stretch by a factor of 3
b) $k = 2$; produces horizontal compression by a factor of $\frac{1}{2}$
c) $d = -5$; produces a vertical translation 5 units down
d) $c = -4$; produces a horizontal translation 4 units left.

2. a) (a) A vertical stretch by a factor of 3 takes $\left(\frac{1}{10}, -1\right)$ to $\left(\frac{1}{10}, -3\right)$, $(1, 0)$ to $(1, 0)$, and $(10, 1)$ to $(10, 3)$.
(b) A horizontal compression by a factor of $\frac{1}{2}$ takes $\left(\frac{1}{10}, -1\right)$ to $\left(\frac{1}{20}, -1\right)$, $(1, 0)$ to $\left(\frac{1}{2}, 0\right)$, and $(10, 1)$ to $(5, 1)$.
(c) A vertical translation 5 units down takes $\left(\frac{1}{10}, -1\right)$ to $\left(\frac{1}{10}, -6\right)$, $(1, 0)$ to $(1, -5)$, and $(10, 1)$ to $(10, -4)$.
(d) A horizontal translation 4 units to the left takes $\left(\frac{1}{10}, -1\right)$ to $(-3\frac{9}{10}, -1)$, $(1, 0)$ to $(-3, 0)$, and $(10, 1)$ to $(6, 1)$.
b) (a) $D = \{x \in \mathbf{R} | x > 0\}$, $R = \{y \in \mathbf{R}\}$
(b) $D = \{x \in \mathbf{R} | x > 0\}$, $R = \{y \in \mathbf{R}\}$
(c) $D = \{x \in \mathbf{R} | x > 0\}$, $R = \{y \in \mathbf{R}\}$
(d) $D = \{x \in \mathbf{R} | x > -4\}$, $R = \{y \in \mathbf{R}\}$

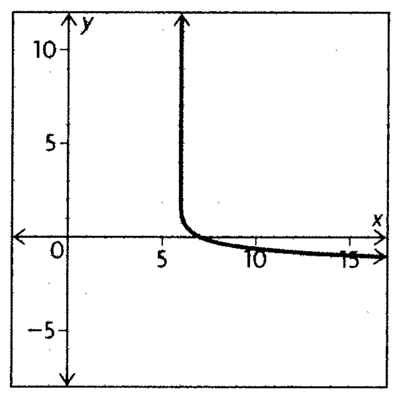
3. a) $a = 5$; $c = 3$; $f(x) = 5 \log_{10} x + 3$
b) $a = -1$ (reflection in the x-axis);
$k = 3$ (compression with a factor of 3);
$f(x) = -\log_{10}(3x)$
c) $c = -3$; $d = -4$; $f(x) = \log_{10}(x + 4) - 3$
d) $a = -1$; $d = 4$; $f(x) = -\log_{10}(x - 4)$

4. i) a) $a = -4$ resulting in a reflection in the x-axis and a vertical stretch by a factor of 4; $c = 5$ resulting in a translation 5 units up.
b) A vertical stretch by a factor of 4 followed by a reflection in the x-axis and a translation 5 units up takes $(1, 0)$ to $(1, 5)$.
A vertical stretch by a factor of 4 followed by a reflection in the x-axis and a translation 5 units up takes $(10, 1)$ to $(10, 1)$.
c) vertical asymptote is $x = 0$
d) $D = \{x \in \mathbf{R} | x > 0\}$, $R = \{y \in \mathbf{R}\}$
ii) a) $a = \frac{1}{2}$ resulting in a vertical compression by a factor of $\frac{1}{2}$; $d = 6$ resulting in a horizontal translation 6 units to the right; $c = 3$ resulting in a vertical translation 3 units up.

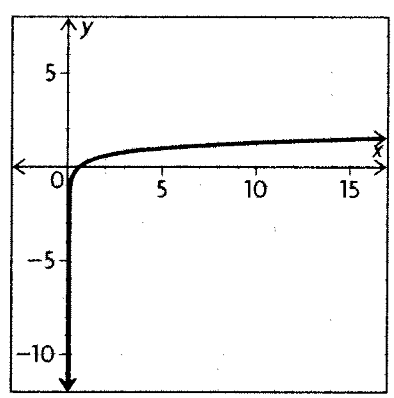
b) A vertical compression by a factor of $\frac{1}{2}$ followed by a translation 6 units to the right and a translation 3 units up takes (1, 0) to (7, 3).
A vertical compression by a factor of $\frac{1}{2}$ followed by a translation 6 units to the right and a translation 3 units up takes (10, 1) to $\left(16, 3\frac{1}{2}\right)$.
c) vertical asymptote is $x = 6$
d) $D = \{x \in \mathbf{R} \mid x > 6\}$, $R = \{y \in \mathbf{R}\}$
iii) a) $k = 3$ resulting in a horizontal compression by a factor of $\frac{1}{3}$; $c = -4$ resulting in a vertical shift 4 units down.
b) A horizontal compression by a factor of $\frac{1}{3}$ followed by a translation 4 units down takes (1, 0) to $\left(\frac{1}{3}, -4\right)$.
A horizontal compression by a factor of $\frac{1}{3}$ followed by a translation 4 units down takes (10, 1) to $\left(3\frac{1}{3}, -3\right)$.
c) vertical asymptote is $x = 0$
d) $D = \{x \in \mathbf{R} \mid x > 0\}$, $R = \{y \in \mathbf{R}\}$
iv) a) $a = 2$ resulting in a vertical stretch by a factor of 2; $k = -2$ resulting in a horizontal compression by a factor of $\frac{1}{2}$ and a reflection in the y-axis; $d = -2$ resulting in a horizontal translation 2 units to the left.
b) A horizontal compression by a factor of $\frac{1}{2}$, a reflection in the y-axis, and a vertical stretch by a factor of 2, followed by a translation 2 units to the left takes (1, 0) to $\left(-1\frac{1}{2}, 0\right)$.
A horizontal compression by a factor of $\frac{1}{2}$, a reflection in the y-axis, and a vertical stretch by a factor of 2, followed by a translation 2 units to the left takes (10, 1) to $(-7, 2)$.
c) vertical asymptote is $x = -2$
d) $D = \{x \in \mathbf{R} \mid x < -2\}$, $R = \{y \in \mathbf{R}\}$
v) a) $\log_{10}(2x + 4) = \log_{10}[2(x + 2)]k = 2$ resulting in a horizontal compression by a factor of $\frac{1}{2}$; $d = -2$ resulting in a horizontal translation 2 units to the left.
b) A horizontal compression by a factor of $\frac{1}{2}$ followed by a translation 2 units to the left takes (1, 0) to $\left(-1\frac{1}{2}, 0\right)$.
A horizontal compression by a factor of $\frac{1}{2}$ followed by a translation 2 units to the left takes (10, 1) to (3, 1).
c) vertical asymptote is $x = -2$
d) $D = \{x \in \mathbf{R} \mid x > -2\}$, $R = \{y \in \mathbf{R}\}$
vi) a) $\log_{10}(-x - 2) = \log_{10}[-1(x + 2)]k = -1$ resulting in a reflection in the x-axis; $d = -2$, resulting in a horizontal translation 2 units to the right.
b) A reflection in the y-axis followed by a translation 2 units to the left takes (1, 0) to $(-3, 0)$
A reflection in the y-axis followed by a translation 2 units to the left takes (10, 1) to $(-12, 1)$.
c) vertical asymptote is $x = -2$
d) $D = \{x \in \mathbf{R} \mid x < -2\}$, $R = \{y \in \mathbf{R}\}$
5. a) vertical stretch by a factor of 3; vertical translation 3 units up
$D = \{x \in \mathbf{R} \mid x > 0\}$, $R = \{y \in \mathbf{R}\}$



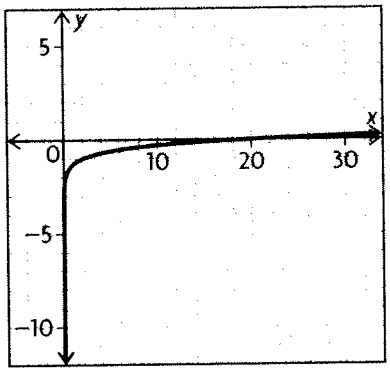
b) reflection in the x-axis; horizontal translation 6 units to the right
$D = \{x \in \mathbf{R} \mid x > -6\}$, $R = \{y \in \mathbf{R}\}$



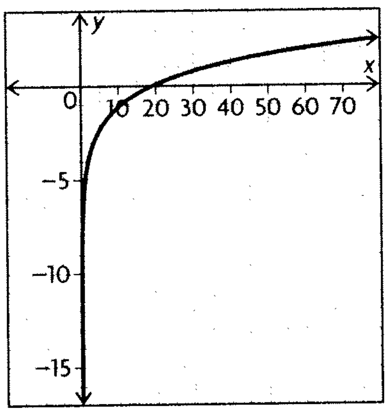
c) horizontal compression by a factor of $\frac{1}{2}$
$D = \{x \in \mathbf{R} \mid x > 0\}$, $R = \{y \in \mathbf{R}\}$



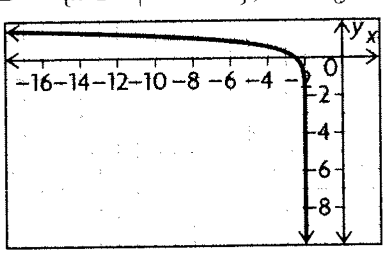
d) horizontal stretch by a factor of 2;
vertical translation 1 unit down
$D = \{x \in \mathbf{R} \mid x > 0\}$, $R = \{y \in \mathbf{R}\}$



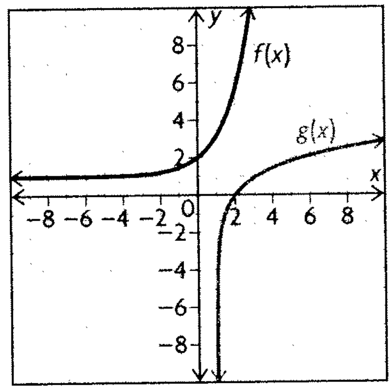
e) vertical stretch by a factor of 4;
horizontal stretch by a factor of 6;
vertical translation 2 units down
$D = \{x \in \mathbf{R} \mid x > 0\}$, $R = \{y \in \mathbf{R}\}$



f) reflection in the y-axis;
vertical translation 2 units up;
horizontal compression by a factor of $\frac{1}{2}$
$D = \{x \in \mathbf{R} \mid x < -2\}$, $R = \{y \in \mathbf{R}\}$



6. Graph the two functions.



It appears from the graph that the functions are inverses of each other. Attempt to verify this.

$$y = 10^{\frac{x}{3}} + 1$$
$$x = 10^{\frac{y}{3}} + 1$$
$$x - 1 = 10^{\frac{y}{3}}$$
$$\log(x - 1) = \frac{y}{3}$$
$$3\log(x - 1) = y$$

This verifies that the functions are inverses of each other.

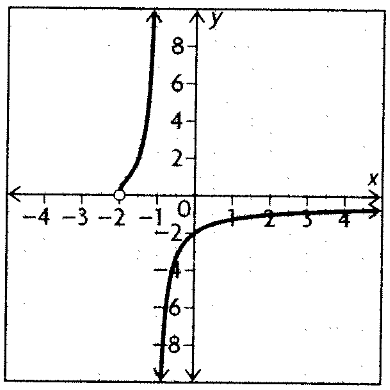
7. a) The graph of $g(x) = \log_3(x + 4)$ is the same as the graph of $f(x) = \log_3 x$, but horizontally translated 4 units to the left. The graph of $h(x) = \log_3 x + 4$ is the same as the graph of $f(x) = \log_3 x$, but vertically translated 4 units up.
b) The graph of $m(x) = 4\log_3 x$ is the same as the graph of $f(x) = \log_3 x$, but vertically stretched by a factor of 4. The graph of $n(x) = \log_3 4x$ is the same as the graph of $f(x) = \log_3 x$, but horizontally compressed by a factor of $\frac{1}{4}$.

8. a) $a = -3$ (vertical stretch and reflection in x-axis); $k = \frac{1}{2}$ (horizontal stretch of 2); $d = 5$ (horizontal translation); $c = 2$ (vertical translation)
$f(x) = -3\log_{10}\left(\frac{1}{2}x - 5\right) + 2$
b) multiply the y coordinate by -3 (for the vertical stretch) and add 2 (for the vertical translation), add 5 to the x-coordinate (for the horizontal translation) and multiply by 2 (for the horizontal stretch).
$(10, 1)$ shifts to $(30, -1)$
c) $D = \{x \in \mathbf{R} \mid x > 5\}$, $R = \{y \in \mathbf{R}\}$

9. Vertical compression by a factor of $\frac{1}{2}$, reflection in the x-axis, horizontal translation 5 units to the left.

10. domain, range, and vertical asymptote

11.



8.3 Evaluating Logarithms, pp. 466–468

1. a) $\log_4 16 = 2$

b) $\log_3 81 = 4$

c) $\log_8 1 = 0$

d) $\log_6 \frac{1}{36} = -2$

e) $\log_{\frac{1}{3}} \frac{1}{27} = 3$

f) $\log_8 2 = \frac{1}{3}$

2. a) $2^3 = 8$

b) $5^{-2} = \frac{1}{25}$

c) $3^4 = 81$

d) $\left(\frac{1}{6}\right)^{-3} = 216$

e) $6^{\frac{1}{2}} = \sqrt{6}$

f) $10^0 = 1$

3. a) $5^x = 5; x = 1$

b) $7^x = 1; x = 0$

c) $2^x = \frac{1}{4}; x = -2$

d) $7^x = \sqrt{7}; 7^x = 7^{\frac{1}{2}}; x = \frac{1}{2}$

e) $\left(\frac{2}{3}\right)^x = \frac{8}{27}; x = 3$

f) $2^x = \sqrt[3]{2}$

$= 2^{\frac{1}{3}}$

$x = \frac{1}{3}$

4. a) $10^x = \frac{1}{10}; 10^x = 10^{-1}; x = -1$

b) $10^x = 1; 10^x = 10^0; x = 0$

c) $10^x = 1\,000\,000; 10^x = 10^6; x = 6$

d) $10^x = 25; x = 1.40$ (typing log (25) into the calculator); $10^{1.40} \doteq 25$

e) $10^{0.25} = 1.78; x = 1.78$

f) $10^{-2} = 0.01; x = 0.01$

5. a) $6^x = \sqrt{6}$

$6^x = 6^{\frac{1}{2}}$

$x = \frac{1}{2}$

b) $\log_5 125; 5^x = 125; x = 3$

$\log_5 25; 5^x = 25; x = 2$

$\log_5 125 - \log_5 25 = 3 - 2 = 1$

c) $\log_3 81; 3^x = 81; x = 4$

$\log_4 64; 4^x = 64; x = 3$

$\log_3 81 + \log_4 64 = 4 + 3 = 7$

d) $\log_2 \frac{1}{4}; 2^x = \frac{1}{4}; x = -2$

$\log_3 1; 3^x = 1; x = 0$

$\log_2 \frac{1}{4} - \log_3 1; -2 - 0 = -2$

e) $5^x = \sqrt[3]{5}; x = \frac{1}{3}$

f) $3^x = \sqrt{27};$

$3^x = \sqrt{3^3}$

$3^x = 3^{\frac{3}{2}}$

$x = \frac{3}{2}$

6. a) $5^3 = x; x = 125$

b) $x^3 = 27; x = 3$

c) $4^x = \frac{1}{64}; 4^x = \frac{1}{4^3}; 4^x = 4^{-3}; x = -3$

d) $\left(\frac{1}{4}\right)^{-2} = x; 4^2 = x; x = 16$

e) $5^{\frac{1}{2}} = x; x = \sqrt{5}$

f) $4^{1.5} = 8$

7. $f(x) = 3^x$ can be written as $\log_3 f(x)$.

a) $f(x) = 17$. The point (2.58, 17) is on the graph. $\log_3 17 \doteq 2.58$.

b) $f(x) = 36$. The point (3.26, 36) is on the graph. $\log_3 36 \doteq 3.26$.

c) $f(x) = 112$. The point (4.29, 112) is on the graph. $\log_3 112 \doteq 4.29$.

d) $f(x) = 143$. The point (4.52, 143) is on the graph. $\log_3 143 \doteq 4.52$.

8. a) $4^x = 32$; by guess and check $x \doteq 2.50$

b) $6^x = 115$; by guess and check $x \doteq 2.65$

c) $3^x = 212$; by guess and check $x \doteq 4.88$

d) $11^x = 896$; by guess and check $x \doteq 2.83$

9. a) $\log_3 3^5; 3^x = 3^5; x = 5$

b) Given that $\log_5 25 = 2$, $5^{\log_5(25)} = 5^2 = 25$

c) Given that $\log_4 \frac{1}{16} = -2$, $4^{\log_4 \frac{1}{16}} = 4^{-2} = \frac{1}{16}$

d) The base m log of a value is the exponent you need to raise m to in order to get that value. You need to raise m to the n power to get m^n. Therefore, $\log_m m^n = n$.

e) The expression $\log_a b$ means what power do you need to raise a to in order to get b. If you substitute that answer into the expression $a^{\log_a b}$ the result is b.

f) $\log_n 1 = 0$ for all non zero values for n.

10. $\log_2 16^{\frac{1}{3}} = \log_2 (2^{(4)})^{(\frac{1}{3})}$

$\log_2 2^{\frac{4}{3}} = \frac{4}{3}$

11. $40(10^x) = 2000$

$10^x = 50$

$\log_{10} 50 = x$

$x \doteq 1.7$ weeks

12. To determine amount of decay, use $\left(\frac{1}{2}\right)^{\frac{n}{1620}}$ where n is the number of years the radium has been decaying.

a) $5\left(\frac{1}{2}\right)^{\frac{150}{1620}} = 4.68\ g$

b) $5\left(\frac{1}{2}\right)^{\frac{n}{1620}} = 4\ g$

$\left(\frac{1}{2}\right)^{\frac{n}{1620}} = \frac{4}{5}$

$\log_{0.5} \frac{4}{5} = \frac{n}{1620}$

$1620 \log_{0.5} \frac{4}{5} = n$

$n = 522$ years

13. A: $s(0.0625) = 0.159 + 0.118 \log(0.0625)$

Slope $= 0.017$

B: $s(1) = 0.159 + 0.118 \log(1)$

Slope $= 0.159$

B has a steeper slope

14. a) $s(50) = 93 \log(50) + 65$

$s(50) \doteq 233$ mph

b) $250 = 93 \log d + 65$

$185 = 93 \log d$

$1.99 = \log d$

$10^{1.99} = d$

$d = 98$ miles

15. If $\log 365 = \frac{3}{2} \log 150 - 0.7$, then Kepler's equation gives a good approximation of the time it takes for Earth to revolve around the sun.

$\log 365 = 2.562$

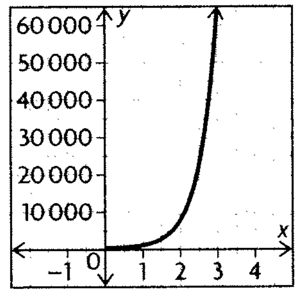
$\frac{3}{2} \log 150 - 0.7 = 2.564$

16. a) $\frac{3}{2} \log 2854 - 0.7 \doteq 83$ years

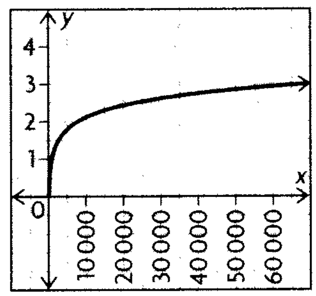
b) $\frac{3}{2} \log 4473 - 0.7 \doteq 164$ years

17. a) $y = 100(2)^{\frac{x}{0.32}}$

b)



c)



d) $y = 0.32 \log_2\left(\frac{x}{100}\right)$; this equation tells how many hours, y, it will take for the number of bacteria to reach x.

e) Evaluate the inverse function for $x = 450$.

$y = 0.32 \log_2\left(\frac{450}{100}\right)$

$y \doteq 0.69$ h

18. a) $\log_5 5 = \frac{\log 5}{\log 5}$

$= 1.0000$

b) $\log_2 10 = \frac{\log 10}{\log 2}$

$= 3.3219$

c) $\log_5 45 = \frac{\log 45}{\log 5}$

$= 2.3652$

d) $\log_8 92 = \frac{\log 92}{\log 8}$

$= 2.1745$

e) $\log_4 0.5 = \frac{\log 0.5}{\log 4}$

$= -0.5000$

f) $\log_7 325 = \frac{\log 325}{\log 7}$

$= 2.9723$

19. a) positive for all values $x > 1$

b) negative for all values $0 < x < 1$

c) undefined for all values $x \le 0$

20. a) $3^3 + 10^3 = 27 + 1000 = 1027$

b) $5^{1.292} - 3^{3.24} = 8 - 35.14 = -27.14$

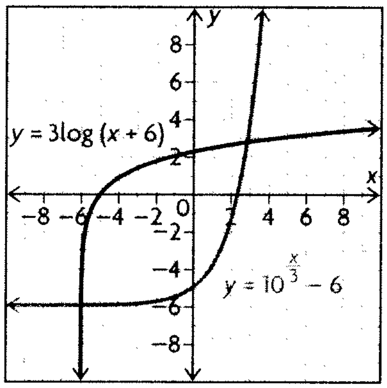
21. a) $y = x^3$

b) $\frac{\sqrt{2}}{3}$

c) $\sqrt[x-2]{0.5}$

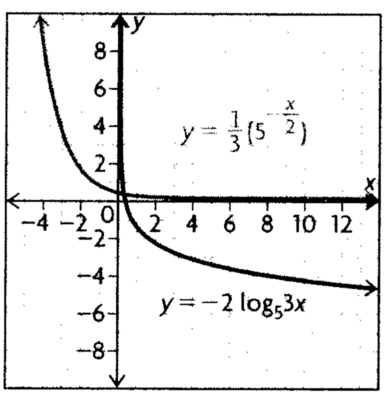
d) $2^{\frac{x-2}{3}} + 3$

22. a)



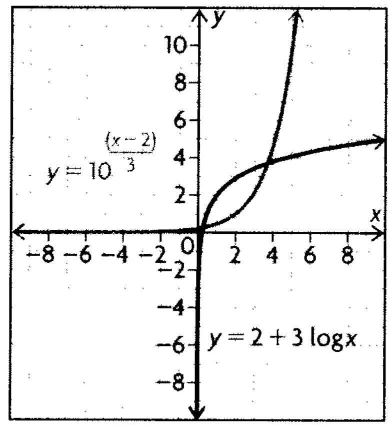
function: $y = 3\log(x + 6)$ $D = \{x \in \mathbf{R} \mid x > -6\}$ $R = \{y \in \mathbf{R}\}$ asymptote: $x = -6$
inverse: $y = 10^{\frac{x}{3}} - 6$ $D = \{x \in \mathbf{R}\}$ $R = \{y \in \mathbf{R} \mid y > -6\}$ asymptote: $y = -6$

b)



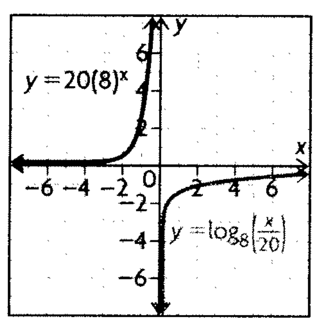
function: $y = -2\log_5 3x$ $D = \{x \in \mathbf{R} \mid x > 0\}$ $R = \{y \in \mathbf{R}\}$ asymptote: $x = 0$
inverse: $y = \frac{1}{3}(5^{-\frac{x}{2}})$ $D = \{x \in \mathbf{R}\}$ $R = \{y \in \mathbf{R} \mid y > 0\}$ asymptote: $y = 0$

c)



function: $y = 2 + 3\log x$ $D = \{x \in \mathbf{R} \mid x > 0\}$ $R = \{y \in \mathbf{R}\}$ asymptote: $x = 0$
inverse: $y = 10^{\frac{(x-2)}{3}}$ $D = \{x \in \mathbf{R}\}$ $R = \{y \in \mathbf{R} \mid y > 0\}$ asymptote: $y = 0$

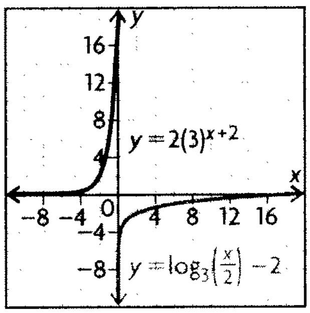
d)



function: $y = 20(8)^x$
$D = \{x \in \mathbf{R}\}$
$R = \{y \in \mathbf{R} | y > 0\}$
asymptote: $y = 0$

inverse: $y = \log_8\left(\frac{x}{20}\right)$
$D = \{x \in \mathbf{R} | x > 0\}$
$R = \{y \in \mathbf{R}\}$
asymptote: $x = 0$

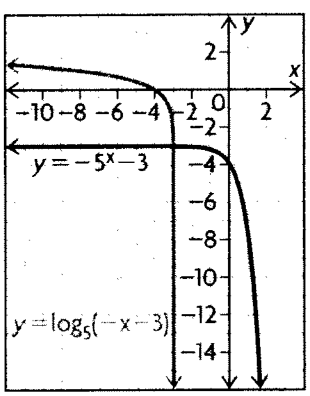
e)



function: $y = 2(3)^{x+2}$
$D = \{x \in \mathbf{R}\}$
$R = \{y \in \mathbf{R} | y > 0\}$
asymptote: $y = 0$

inverse: $y = \log_3\left(\frac{x}{2}\right) - 2$
$D = \{x \in \mathbf{R} | x > 0\}$
$R = \{y \in \mathbf{R}\}$
asymptote: $x = 0$

f)



function: $y = -5^x - 3$
$D = \{x \in \mathbf{R}\}$
$R = \{y \in \mathbf{R} | y < -3\}$
asymptote: $y = -3$

inverse:
$y = \log_5(-x - 3)$
$D = \{x \in \mathbf{R} | x < -3\}$
$R = \{y \in \mathbf{R}\}$
asymptote: $x = -3$

23. Given the constraints, two integer values are possible for y, either 1 or 2. If $y = 3$, then x must be 1000, which is not permitted.

8.4 Laws of Logarithms, pp. 475–476

1. a) $\log 45 + \log 68$
b) $\log_m p + \log_m q$
c) $\log 123 - \log 31$
d) $\log_m p - \log_m q$
e) $\log_2 14 + \log_2 9$
f) $\log_4 81 - \log_4 30$
2. a) $\log(7 \times 5) = \log 35$
b) $\log_3 \frac{4}{2} = \log_3 2$
c) $\log_m ab$
d) $\log \frac{x}{y}$
e) $\log_6(7 \times 8 \times 9) = \log_6 504$
f) $\log_4\left(\frac{(10 \times 12)}{20}\right) = \log_4 6$
3. a) $2 \log 5$
b) $-1 \log 7$
c) $q \log_m p$
d) $\frac{1}{3} \log 45$
e) $\frac{1}{2} \log_7 36$
f) $\frac{1}{5} \log_5 125$
4. a) $\log_3 135 - \log_3 5 = \log_3 \frac{135}{5}$
$= \log_3 27$
$= 3$
b) $\log_5 10 + \log_5 2.5 = \log_5(10 \times 2.5)$
$= \log_5 25$
$= 2$

c) $\log 50 + \log 2 = \log(50 \times 2)$
$= \log 100$
$= 2$

d) $\log_4 4^7 = 7\log_4 4$
$= 7 \times 1$
$= 7$

e) $\log_2 224 - \log_2 7 = \log_2 \frac{224}{7}$
$= \log_2 32$
$= 5$

f) $\log \sqrt{10} = \frac{1}{2}\log 10$
$= \left(\frac{1}{2}\right)(1)$
$= \frac{1}{2}$

5. $y = \log_2(4x) = \log_2 x + \log_2 4 = \log_2 x + 2$, so $y = \log_2(4x)$ vertically shifts $y = \log_2 x$ up 2 units; $y = \log_2(8x) = \log_2 x + \log_2 8 = \log_2 x + 3$, so $y = \log_2(8x)$ vertically shifts $y = \log_2 x$ up 3 units; $y = \log_2\left(\frac{x}{2}\right) = \log_2 x - \log_2 2 = \log_2 x - 1$, so $y = \log_2\left(\frac{x}{2}\right)$ vertically shifts $y = \log_2 x$ down 1 unit

6. a) $\log_{25} 5^3 = 3\log_{25} 5$
$\log_{25} 5$; $25^x = 5$; $x = 0.5$
Therefore $\log_{25} 5^3 = 3\log_{25} 5 = (3)(0.5) = 1.5$

b) $\log_6 54 + \log_6 2 - \log_6 3 = \log_6 \frac{(54 \times 2)}{3}$
$= \log_6 36$
$= 2$

c) $\log_6 6\sqrt{6} = \log_6 6 + \log_6 \sqrt{6}$
$= 1 + 0.5$
$= 1.5$

d) $\log_2 \sqrt{36} - \log_2 \sqrt{72} = \log_2 \frac{\sqrt{36}}{\sqrt{72}}$
$= \log_2 \sqrt{\frac{1}{2}}$
$= \log_2 2^{-0.5}$
$= -0.5$

e) $\log_3 54 + \log_3\left(\frac{3}{2}\right) = \log_3 54 + \log_3 3 - \log_3 2$
$= \log_3 54 - \log_3 2 + 1$
$= \log_3 \frac{54}{2} + 1$
$= \log_3 27 + 1$
$= 3 + 1$
$= 4$

f) $\log_8 2 + 3\log_8 2 + \frac{1}{2}\log_8 16$
$= \log_8 2 + \log_8 2^3 + \log_8 \sqrt{16}$
$= \log_8 2 + \log_8 8 + \log_8 4$
$= \log_8 2 + \log_8 4 + 1$
$= \log_8(2 \times 4) + 1$
$= \log_8 8 + 1$
$= 1 + 1$
$= 2$

7. a) $\log_b x + \log_b y + \log_b z$
b) $\log_b z - (\log_b x + \log_b y)$
c) $\log_b x^2 + \log_b y^3 = 2\log_b x + 3\log_b y$
d) $\log_b \sqrt{x^5yz^3} = \frac{1}{2}\log_b x^5yz^3$
$= \frac{1}{2}(\log_b x^5 + \log_b y + \log_b z^3)$
$= \frac{1}{2}(5\log_b x + \log_b y + 3\log_b z)$

8. $\log_5 3$ means $5^x = 3$ and $\log_5 \frac{1}{3}$ means $5^y = \frac{1}{3}$; since $\frac{1}{3} = 3^{-1}$, $5^y = 5^{x(-1)}$; therefore, $\log_5 3 + \log_5 \frac{1}{3} = x + x(-1) = 0$

9. a) $3\log_5 2 + \log_5 7 = \log_5 2^3 + \log_5 7$
$= \log_5 8 + \log_5 7$
$= \log_5(8 \times 7)$
$= \log_5 56$

b) $2\log_3 8 - 5\log_3 2 = \log_3 8^2 - \log_3 2^5$
$= \log_3 64 - \log_3 32$
$= \log_3 \frac{64}{32}$
$= \log_3 2$

c) $2\log_2 3 + \log_2 5 = \log_2 3^2 + \log_2 5$
$= \log_2 9 + \log_2 5$
$= \log_2(9 \times 5)$
$= \log_2 45$

d) $\log_3 12 + \log_3 2 - \log_3 6 = \log_3 \frac{(12 \times 2)}{6}$
$= \log_3 4$

e) $\log_4 3 + \frac{1}{2}\log_4 8 - \log_4 2$
$= \log_4 3 + \log_4 \sqrt{8} - \log_4 2$
$= \log_4 \frac{(3\sqrt{8})}{2}$
$= \log_4 (3\sqrt{2})$

f) $2\log 8 + \log 9 - \log 36$
$= \log 8^2 + \log 9 - \log 36$
$= \log 64 + \log 9 - \log 36$
$= \log \frac{(64 \times 9)}{36}$
$= \log 16$

10. a) $\log_2 x = \log_2 7^2 + \log_2 5$
$\log_2 x = \log_2 49 + \log_2 5$
$\log_2 x = \log_2 (49 \times 5)$
$\log_2 x = \log_2 245$
$x = 245$

b) $\log x = \log 4^2 + \log 3^3$
$\log x = \log 16 + \log 27$
$\log x = \log (16 \times 27)$
$\log x = \log 432$
$x = 432$

c) $\log_4 x = \log_4 48 - \log_4 12$
$\log_4 x = \log_4 \frac{48}{12}$
$\log_4 x = \log_4 4$
$x = 4$

d) $\log_7 x = \log_7 25^2 - \log_7 5^3$
$\log_7 x = \log_7 625 - \log_7 125$
$\log_7 x = \log_7 \frac{625}{125}$
$\log_7 x = \log_7 5$
$x = 5$

e) $\log_3 x = \log_3 10^2 - \log_3 25$
$\log_3 x = \log_3 100 - \log_3 25$
$\log_3 x = \log_3 \frac{100}{25}$
$\log_3 x = \log_3 4$
$x = 4$

f) $\log_5 x = \log_5 6 + 3\log_5 2 + \log_5 8$
$\log_5 x = \log_5 6 + \log_5 2^3 + \log_5 8$
$\log_5 x = \log_5 6 + \log_5 8 + \log_5 8$
$\log_5 x = \log_5 (6 \times 8 \times 8)$
$\log_5 x = \log_5 384$
$x = 384$

11. a) $\log_2 xyz$

b) $\log_5 \frac{uw}{v}$

c) $\log_6 a - \log_6 bc = \log_6 \frac{a}{bc}$

d) $\log_2 \frac{x^2y^2}{xy} = \log_2 xy$

e) $\log_3 3 + \log_3 x^2 = \log_3 3x^2$

f) $\log_4 x^3 + \log_4 x^2 - \log_4 y = \log_4 \frac{x^3x^2}{y}$
$= \log_4 \frac{x^5}{y}$

12. $\frac{1}{2}\log_a x = \log_a \sqrt{x}$; $\frac{1}{2}\log_a y = \log_a \sqrt{y}$;
$\frac{3}{4}\log_a z = \log_a \sqrt[4]{z^3}$;

So, $\frac{1}{2}\log_a x + \frac{1}{2}\log_a y - \frac{3}{4}\log_a x$
$= \log_a \sqrt{x} + \log_a \sqrt{y} - \log_a \sqrt[4]{z^3}$
$= \log_a \frac{\sqrt{x}\sqrt{y}}{\sqrt[4]{z^3}}$

13. vertical stretch by a factor of 3 (the exponent that x is raised to); vertical shift 3 units up (the coefficient 8 divided by the base of 2)

14. Answers may vary. For example,
$f(x) = 2\log x - \log 12$
$g(x) = \log \frac{x^2}{12}$
$2\log x - \log 12 = \log x^2 - \log 12$
$= \log \frac{x^2}{12}$
So, $f(x)$ and $g(x)$ have the same graph.

15. Answers may vary. For example, any number can be written as a power with a given base. The base of the logarithm is 3. Write each term in the quotient as a power of 3. The laws of logarithms make it possible to evaluate the expression by simplifying the quotient and noting the exponent.

16. $\log_x x^{m-1} + 1 = m - 1 + 1 = m$

17. $\log_b x\sqrt{x} = \log_b x + \log_b \sqrt{x}$
$= \log_b x + \frac{1}{2}\log_b x$
$= 0.3 + 0.3\left(\frac{1}{2}\right)$
$= 0.45$

18. The two functions have different domains. The first function has a domain of $x > 0$. The second function has a domain of all real numbers except 0, since x is squared.

19. Answers may vary; for example,
Product law
$\log_{10} 10 + \log_{10} 10 = 1 + 1$
$= 2$
$= \log_{10} 100$
$= \log_{10} (10 \times 10)$

Quotient law
$\log_{10} 10 - \log_{10} 10 = 1 - 1$
$= 0$
$= \log_{10} 1$
$= \log_{10}\left(\frac{10}{10}\right)$

Power law
$\log_{10} 10^2 = \log_{10} 100$
$= 2$
$= 2\log_{10} 10$

Mid-Chapter Review, p. 479

1. a) $\log_5 y = x$
b) $\log_{\frac{1}{3}} y = x$
c) $\log x = y$
d) $\log_p m = q$
2. a) $3^y = x$
b) $10^y = x$
c) $10^k = m$
d) $s^t = r$
3. a) $a = 2$; $c = -4$; vertical stretch by a factor of 2, vertical translation 4 units down
b) $a = -1$; $k = 3$; reflection in the x-axis, horizontal compression by a factor of $\frac{1}{3}$
c) $d = -5$; $c = 1$; vertical compression by a factor of $\frac{1}{4}$, horizontal stretch by a factor of 4
d) $k = 2$; $d = 2$; horizontal compression by a factor of $\frac{1}{2}$, horizontal translation 2 units to the right
e) $a = \frac{1}{4}$; $k = \frac{1}{4}$; horizontal translation 5 units to the left, vertical translation 1 unit up
f) $a = 5$; $k = -1$; $c = -3$; vertical stretch by a factor of 5, reflection in the y-axis, vertical translation 3 units down
4. a) $y = -4\log_3 x$
b) $y = \log_3(x + 3) + 1$
c) $y = \frac{2}{3}\log_3\left(\frac{1}{2}x\right)$
d) $y = 3\log_3[-(x - 1)]$
5. a) A vertical stretch by a factor of 4 followed by a reflection in the x-axis takes (9, 2) to (9, −8).
b) A horizontal translation 3 units to the left followed by a translation 1 unit up takes (9, 2) to (6, 3).
c) A vertical compression by a factor of $\frac{2}{3}$ followed by a horizontal stretch by a factor of 2 takes (9, 2) to $(18, \frac{4}{3})$.
d) A vertical stretch by a factor of 3 followed by a reflection in the y-axis and a horizontal translation 1 unit to the right takes (9, 2) to (9, 6), then to (−9, 6), and finally to (−8, 6).
6. It is vertically stretched by a factor of 2 and vertically shifted up 2.
7. a) $3^x = 81; x = 4$
b) $4^x = \frac{1}{16}; x = -2$
c) $5^x = 1; x = 0$
d) $\frac{2^x}{3} = \frac{27}{8}; x = -3$
8. Using the base 10 log key on the calculator
a) 0.602
b) 1.653
c) 2.130
d) 2.477
9. Using a guess and check strategy on the graphing calculator to estimate the result
a) $2^x = 21; x \doteq 4.392$
b) $5^x = 117; x \doteq 2.959$
c) $7^x = 141; x \doteq 2.543$
d) $11^x = 356; x \doteq 2.450$
10. a) $\log(7 \times 4) = \log 28$
b) $\log\frac{5}{2} = \log 2.5$
c) $\log_3\frac{(11 \times 4)}{6} = \log_3\frac{22}{3}$
d) $\log_p(q \times q) = \log_p q^2$
11. a) $\log_{11}\frac{33}{3} = \log_{11} 11 = 1$
b) $\log_7(14 \times 3.5) = \log_7 49 = 2$
c) $\log_5\left(100 \times \frac{1}{4}\right) = \log_5 25 = 2$
d) $\log_{\frac{1}{2}}\frac{72}{9} = \log_{\frac{1}{2}} 8$

$$\left(\frac{1}{2}\right)^x = 8$$
$$x = -3$$

e) $\frac{1}{3}\log_4 16 = \left(\frac{1}{3}\right)(2) = \frac{2}{3}$
f)
$$\begin{aligned}\log_3 9\sqrt{27} &= \log_3 9 + \log_3\sqrt{27}\\ &= 2 + \frac{1}{2}\log_3 27\\ &= 2 + \left(\frac{1}{2}\right)(3)\\ &= 3.5\end{aligned}$$
12. Compared with the graph of $y = \log x$, the graph of $y = \log x^3$ is vertically stretched by a factor of 3.
13. Use the log button on a calculator to evaluate each expression.
a) $\log 4^8 = 4.82$
b) $\log 200 \div \log 50 = 1.35$
c) $\log\sqrt{40} = 0.80$
d) $(\log 20)^2 = 1.69$
e) $\log 9^4 = 3.82$
f) $5\log 5 = 3.49$

8.5 Solving Exponential Equations, pp. 485–486

1. a) $x = 4$

b) $(2^2)^{2x} = 2^{5-x}$
$2^{4x} = 2^{5-x}$
$4x = 5 - x$
$5x = 5$
$x = 1$

c) $(3^2)^{x+1} = (3^3)^{2x-3}$
$3^{2x+2} = 3^{6x-9}$
$2x + 2 = 6x - 9$
$11 = 4x$
$x = \frac{11}{4}$

d) $(2^3)^{x-1} = (2^4)^{\frac{1}{3}}$
$2^{3x-3} = 2^{\frac{4}{3}}$
$3x - 3 = \frac{4}{3}$
$9x - 9 = 4$
$9x = 13$
$x = \frac{13}{9}$

e) $2^{3x} = 2^{-1}$
$3x = -1$
$x = -\frac{1}{3}$

f) $4^{2x} = 4^{-2}$
$2x = -2$
$x = -1$

2. a) $\log 2^x = \log 17$
$x \log 2 = \log 17$
$0.301x = 1.230$
$x = 4.088$

b) $\log 6^x = \log 231$
$x \log 6 = \log 231$
$0.778x = 2.364$
$x = 3.037$

c) $5^x = 5$
$x = 1$

d) $5.25 = 1.5^x$
$\log 1.5^x = \log 5.25$
$x \log 1.5 = \log 5.25$
$0.176x = 0.720$
$x = 4.092$

e) $\log 5^{1-x} = \log 10$
$(1 - x) \log 5 = \log 10$
$(1 - x)(0.699) = 1$
$0.699 - 0.699x = 1$
$-0.699x = 0.301$
$x = -0.431$

f) $\log 6^{\frac{x}{3}} = \log 30$
$\frac{x}{3} \log 6 = \log 30$
$\frac{x}{3}(0.778) = 1.477$
$x = 5.695$

3. a) $3^x = 243$
$3^x = 3^5$
$x = 5$

b) $6^x = 216$
$6^x = 6^3$
$x = 3$

c) $5^x = 5\sqrt{5}$
$5^x = 5^{1.5}$
$x = 1.5$

d) $2^x = \sqrt[5]{8}$
$2^x = \sqrt[5]{2^3}$
$2^x = 2^{\frac{3}{5}}$
$x = \frac{3}{5}$

e) $2^x = \frac{1}{4}$
$2^x = 2^{-2}$
$x = -2$

f) $3^x = \frac{1}{\sqrt{3}}$
$3^x = 3^{-\frac{1}{2}}$
$x = -\frac{1}{2}$

4. a) $200 = 300\left(\frac{1}{2}\right)^{\frac{t}{8}}$
$\frac{2}{3} = \left(\frac{1}{2}\right)^{\frac{t}{8}}$
$\log \frac{2}{3} = \log\left(\frac{1}{2}\right)^{\frac{t}{8}}$
$\log \frac{2}{3} = \frac{t}{8} \log\left(\frac{1}{2}\right)$
$-0.176 = \frac{t}{8}(-0.301)$
$t = 4.68 \text{ h}$

b) $100 = 300\left(\frac{1}{2}\right)^{\frac{t}{8}}$

$\frac{1}{3} = \left(\frac{1}{2}\right)^{\frac{t}{8}}$

$\log \frac{1}{3} = \log\left(\frac{1}{2}\right)^{\frac{t}{8}}$

$\log \frac{1}{3} = \frac{t}{8} \log\left(\frac{1}{2}\right)$

$-0.477 = \frac{t}{8}(-0.301)$

$t = 12.68$ h

c) $75 = 300\left(\frac{1}{2}\right)^{\frac{t}{8}}$

$\frac{1}{4} = \left(\frac{1}{2}\right)^{\frac{t}{8}}$

$\log \frac{1}{4} = \log\left(\frac{1}{2}\right)^{\frac{t}{8}}$

$\log \frac{1}{4} = \frac{t}{8} \log\left(\frac{1}{2}\right)$

$-0.602 = \frac{t}{8}(-0.301)$

$t = 16$ h

d) $20 = 300\left(\frac{1}{2}\right)^{\frac{t}{8}}$

$\frac{1}{15} = \left(\frac{1}{2}\right)^{\frac{t}{8}}$

$\log \frac{1}{15} = \log\left(\frac{1}{2}\right)^{\frac{t}{8}}$

$\log \frac{1}{15} = \frac{t}{8} \log\left(\frac{1}{2}\right)$

$-1.176 = \frac{t}{8}(-0.301)$

$t = 31.26$ h

5. a) $(7^2)^{x-1} = 7^{1.5}$

$7^{2x-2} = 7^{1.5}$

$2x - 2 = 1.5$

$2x = 3.5$

$x = 1.75$

b) $2^{3x-4} = 2^{-2}$

$3x - 4 = -2$

$x = \frac{2}{3}$

c) $(2^{-2})^{x+4} = 2^{\frac{3}{2}}$

$2^{-2x-8} = 2^{\frac{3}{2}}$

$-2x - 8 = \frac{3}{2}$

$-4x - 16 = 3$

$-4x = 19$

$x = -4.75$

d) $36^{2x+4} = 36^x$

$2x + 4 = x$

$x = -4$

e) $2^{2x+2} = 64$

$2^{2x+2} = 2^6$

$2x + 2 = 6$

$x = 2$

f) $(3^2)^{2x+1} = (3^4)(3^3)^x$

$3^{4x+2} = 3^{3x+4}$

$4x + 2 = 3x + 4$

$x = 2$

6. a) $1000 = 500(1.08)^t$

$2 = 1.08^t$

$\log 2 = \log 1.08^t$

$\log 2 = t \log 1.08$

$0.301 = 0.033t$

$t = 9.12$ years

b) $5000 = 1000(1.01)^t$

$5 = 1.01^t$

$\log 5 = \log 1.01^t$

$\log 5 = t \log 1.01$

$0.699 = 0.0043t$

$t = 162.6$ months $= 13.5$ years

c) $7500 = 5000(1.025)^t$

$1.5 = 1.025^t$

$\log 1.5 = \log 1.025^t$

$\log 1.5 = t \log 1.025$

$0.176 = 0.0107t$

$t = 16.44$ quarters or 4.1 years

d) $1500 = 500(1.0023)^t$

$3 = 1.0023^t$

$\log 3 = \log 1.0023^t$

$\log 3 = t \log 1.0023$

$0.477 = 0.000998t$

$t = 477.9$ weeks or 9.2 years

7. $20(2^t) = 163\,840$

$2^t = 8192$

$\log 2^t = \log 8192$

$t \log 2 = \log 8192$

$0.301t = 3.913$

$t = 13$ quarter hours or 3.25 h

8. a) $4^x(4 + 1) = 160$

$4^x = 32$

$\log 4^x = \log 32$

$x \log 4 = \log 32$

$0.602x = 1.505$
$x = 2.5$

b) $2^x(2^2 + 1) = 320$
$2^x = 64$
$\log 2^x = \log 64$
$x \log 2 = \log 64$
$0.301x = 1.806$
$x = 6$

c) $2^x(2^2 - 1) = 96$
$2^x = 32$
$\log 2^x = \log 32$
$x \log 2 = \log 32$
$0.301x = 1.505$
$x = 5$

d) $10^x(10 - 1) = 9000$
$10^x = 1000$
$\log 10^x = \log 1000$
$x \log 10 = \log 1000$
$x = 3$

e) $3^x(3^2 + 1) = 30$
$3^x = 3$
$x = 1$

f) $4^x(4^3 - 1) = 63$
$4^x = 1$
$x = 0$

9. a) Solve using logarithms. Both sides can be divided by 225, leaving only a term with a variable in the exponent on the left. This can be solved using logarithms.
b) Solve by factoring out a power of 3 and then simplifying. Logarithms may still be necessary in a situation like this, but the factoring must be done first because logarithms cannot be used on the equation in its current form.

10. a) $\log 5^{t-1} = \log 3.92$
$(t - 1)\log 5 = \log 3.92$
$0.699(t - 1) = 0.593$
$t - 1 = 0.849$
$t = 1.849$

b) $3^x = 25$
$\log 3^x = \log 25$
$x \log 3 = \log 25$
$0.477x = 1.398$
$x = 2.931$

c) $\log 4^{2x} = \log 5^{2x-1}$
$2x \log 4 = (2x - 1)\log 5$
$0.602(2x) = 0.699(2x - 1)$
$1.204x = 1.398x - 0.699$
$-0.194x = -0.699$
$x = 3.606$

d) $2^x = 53.2$
$\log 2^x = \log 53.2$
$x \log 2 = \log 53.2$
$0.301x = 1.726$
$x = 5.734$

11. a) $I_f = I_o(0.95)^t$ where I_f is the final intensity, I_o is the original intensity, and t is the thickness
b) $0.6 = 1(0.95)^t$
$\log 0.6 = \log 0.95^t$
$\log 0.6 = t \log 0.95$
$-0.222 = -0.0222t$
$t = 10$ mm

12. $3^{2x} - 5(3^x) + 6 = 0$
$(3^x - 3)(3^x - 2) = 0$
$3^x - 3 = 0$ or $3^x - 2 = 0$
$3^x = 3$ $\quad$ $3^x = 2$
$x = 1$ $\quad$ $\log 3^x = \log 2$
$x \log 3 = \log 2$
$0.477x = 0.301$
$x = 0.631$

13. $a^y = x$, so $\log a^y = \log x$; $y \log a = \log x$;
$y = \frac{\log x}{\log a}$
A graphing calculator does not allow logarithms of base 5 to be entered directly. However, $y = \log_5 x$ can be entered for graphing, as $y = \frac{\log x}{\log 5}$.

14. a) $\log (2^x)^2 = \log [32(2^{4x})]$
$\log 2x + \log 2x = \log 32 + \log 2^{4x}$
$x \log 2 + x \log 2 = \log 32 + 4x \log 2$
$0.301x + 0.301x = 1.505 + 4x(0.301)$
$x = 2.5$

b) $3^{x^2+20} = (3^{-3})^{3x}$
$x^2 + 20 = -9x$
$x^2 + 9x + 20 = 0$
$(x - 5)(x - 4) = 0$
$x = 5$ or $x = 4$

c) $\log 2 + \log 3^x = \log 7 + \log 5^x$
$\log 2 + x \log 3 = \log 7 + x \log 5$
$0.301 + 0.477x = 0.845 + 0.699x$
$-0.222x = 0.544$
$x = -2.45$

15. Let $\log_a 2 = x$. Then $a^x = 2$. $(a^x)^3 = 2^3$, or $a^{3x} = 8$. Since $\log_a 2 = \log_b 8$, $\log_b 8 = x$. So $b^x = 8$. Since each equation is equal to 8, $a^{3x} = b^x$ and $a^3 = b$.

16. $3(5^{2x}) = 6(4^{3x})$
$5^{2x} = 2(4^{3x})$
$\log 5^{2x} = \log 2(4^{3x})$
$\log 5^{2x} = \log 2 + \log (4^{3x})$
$2x \log 5 = \log 2 + 3x \log 4$

$$2x(0.699) = 0.301 + 3x(0.602)$$
$$1.398x = 0.301 + 1.806x$$
$$x = -0.737$$

Substitute to determine y

$y = 3(5^{(2\times(-0.737))})$

$y = 3(5^{-1.475})$

$y = 0.279$

17. a) $\log 6^{3x} = \log 4^{2x-3}$
$$3x \log 6 = (2x - 3) \log 4$$
$$3x(0.778) = (2x - 3)(0.602)$$
$$2.33x = 1.204x - 1.806$$
$$1.13x = -1.806$$
$$x = -1.60$$

b) $\log (1.2)^x = \log (2.8)^{x+4}$
$$x \log 1.2 = (x + 4) \log 2.8$$
$$0.079x = (x + 4)(0.447)$$
$$0.079x = 0.447x + 1.789$$
$$-0.368x = 1.789$$
$$x = -4.86$$

c) $\log 3(2)^x = \log 4^{x+1}$
$$\log 3 + \log 2^x = (x + 1) \log 4$$
$$\log 3 + x \log 2 = (x + 1) \log 4$$
$$0.477 + 0.301x = (x + 1)(0.602)$$
$$0.477 + 0.301x = 0.602x + 0.602$$
$$-0.125 = 0.301x$$
$$x = -0.42$$

18. $(2^x)^x = 10$
$$2^{x^2} = 10$$
$$\log 2^{x^2} = \log 10$$
$$x^2 \log 2 = 1$$
$$x^2 = \frac{1}{\log 2}$$
$$x = \pm\sqrt{\frac{1}{\log 2}}$$
$$x = \pm 1.82$$

8.6 Solving Logarithmic Equations, pp. 491–492

1. a) $\log_2 x = \log_2 5^2$
$$x = 25$$

b) $\log_3 x = \log_3 3^4$
$$x = 81$$

c) $\log x = \log 2^3$
$$x = 8$$

d) $\log (x - 5) = \log 10$
$$x - 5 = 10$$
$$x = 15$$

e) $2^x = 8$
$$x = 3$$

f) $\log_2 x = \log_2 \sqrt{3}$
$$x = \sqrt{3}$$

2. a) $x^4 = 625$
$$x^4 = 5^4$$
$$x = 5$$

b) $x^{-\frac{1}{2}} = 6$
$$\frac{1}{\sqrt{x}} = 6$$
$$1 = 6\sqrt{x}$$
$$\frac{1}{6} = \sqrt{x}$$
$$\frac{1}{36} = x$$

c) $5^2 = 2x - 1$
$$25 = 2x - 1$$
$$26 = 2x$$
$$x = 13$$

d) $10^3 = 5x - 2$
$$1000 = 5x - 2$$
$$1002 = 5x$$
$$x = 200.4$$

e) $x^{-2} = 0.04$
$$\frac{1}{x^2} = 0.04$$
$$1 = 0.04x^2$$
$$25 = x^2$$
$$x = 5$$

f) $2x - 4 = 36$
$$2x = 40$$
$$x = 20$$

3. $6.3 = \log \frac{a}{1.6} + 4.2$
$$2.1 = \log \frac{a}{1.6}$$
$$10^{2.1} = \frac{a}{1.6}$$
$$125.89 = \frac{a}{1.6}$$
$$201.43 = a$$

4. a) $x^{\frac{3}{2}} = 3^3$
$$x^3 = (3^3)^2$$
$$x^3 = (3^2)^3$$
$$x^3 = 9^3$$
$$x = 9$$

b) $x^2 = 5$
$$x = \sqrt{5}$$

c) $3^3 = 3x + 2$
$27 = 3x + 2$
$25 = 3x$
$x = \frac{25}{3}$

d) $10^4 = x$
$x = 10\,000$

e) $\left(\frac{1}{3}\right)^x = 27$
$3^{-x} = 3^3$
$-x = 3$
$x = -3$

f) $\left(\frac{1}{2}\right)^{-2} = x$
$x = 4$

5. a) $\log_2 3x = 3$
$2^3 = 3x$
$8 = 3x$
$x = \frac{8}{3}$

b) $\log 3x = 1$
$10 = 3x$
$x = \frac{10}{3}$

c) $\log_5 2x + \log_5 \sqrt{9} = 2$
$\log_5 [(3)(2x)] = 2$
$\log_5 6x = 2$
$5^2 = 6x$
$25 = 6x$
$x = \frac{25}{6}$

d) $\log_4 \frac{x}{2} = 2$
$4^2 = \frac{x}{2}$
$16 = \frac{x}{2}$
$x = 32$

e) $\log x^3 - \log 3 = \log 9$
$\log \frac{x^3}{3} = \log 9$
$\frac{x^3}{3} = 9$
$x^3 = 27$
$x^3 = 3^3$
$x = 3$

f) $\log_3 \left[\frac{(4x)(5)}{2}\right] = 4$
$\log_3 10x = 4$
$3^4 = 10x$
$81 = 10x$
$x = 8.1$

6. $\log_6 [x(x - 5)] = 2$
$\log_6 (x^2 - 5x) = 2$
$6^2 = x^2 - 5x$
$36 = x^2 - 5x$
$x^2 - 5x - 36 = 0$
$(x - 9)(x + 4) = 0$
$x = 9$ or $x = -4$
Restrictions: $x > 5$ ($x - 5$ must be positive) so $x = 9$

7. a) $\log_7 [(x + 1)(x - 5)] = 1$
$\log_7 (x^2 - 4x - 5) = 1$
$7^1 = x^2 - 4x - 5$
$x^2 - 4x - 12 = 0$
$(x - 6)(x + 2) = 0$
$x = 6$ or $x = -2$
As x must be >5, -2 is inadmissible. $x = 6$

b) $\log_3 [(x - 2)x] = 1$
$\log_3 (x^2 - 2x) = 1$
$3^1 = x^2 - 2x$
$x^2 - 2x - 3 = 0$
$(x - 3)(x + 1) = 0$
$x = 3$ or $x = -1$
As x must be >2, -1 is inadmissible. $x = 3$

c) $\log_6 \frac{x}{(x - 1)} = 1$
$6^1 = \frac{x}{(x - 1)}$
$6x - 6 = x$
$5x = 6$
$x = \frac{6}{5}$

d) $\log [(2x + 1)(x - 1)] = \log 9$
$2x^2 - x - 1 = 9$
$2x^2 - x - 10 = 0$
$(2x - 5)(x + 2) = 0$
$x = 2.5$ or $x = -2$
As x must be >1, -2 is inadmissible. $x = 2.5$

e) $\log [(x + 2)(x - 1)] = 1$
$10^1 = x^2 + x - 2$
$x^2 + x - 12 = 0$
$(x + 4)(x - 3) = 0$
$x = -4$ or $x = 3$
As x must be >1, -4 is inadmissible. $x = 3$

f) $\log_2 x^3 - \log_2 x = 8$

$$\log_2 \frac{x^3}{x} = 8$$
$$\log_2 x^2 = 8$$
$$2^8 = x^2$$
$$(2^4)^2 = x^2$$
$$x = 2^4 = 16$$

8. a) Use the rules of logarithms to obtain $\log_9 20 = \log_9 x$. Then, because both sides of the equation have the same base, $20 = x$.

b) Use the rules of logarithms to obtain $\log \frac{x}{2} = 3$. Then use the definition of a logarithm to obtain $10^3 = \frac{x}{2}$; $1000 = \frac{x}{2}$; $2000 = x$.

c) Use the rules of logarithms to obtain $\log x = \log 64$. Then, because both sides of the equation have the same base, $x = 64$.

9. a) $50 = 10 \log\left(\frac{I}{10^{-12}}\right)$

$$5 = \log\left(\frac{I}{10^{-12}}\right)$$
$$5 = \log I - \log 10^{-12}$$
$$5 = \log I - (-12)$$
$$-7 = \log I$$
$$10^{-7} = I$$

b) $84 = 10 \log\left(\frac{I}{10^{-12}}\right)$

$$8.4 = \log\left(\frac{I}{10^{-12}}\right)$$
$$8.4 = \log I - \log 10^{-12}$$
$$8.4 = \log (I + 12)$$
$$-3.6 = \log I$$
$$10^{-3.6} = I$$

10. $\log_a \left[\frac{x + 2}{x - 1}\right] = \log_a (8 - 2x)$

$$\frac{x + 2}{x - 1} = 8 - 2x$$
$$x + 2 = -2x^2 + 10x - 8$$
$$-2x^2 + 9x - 10 = 0$$
$$2x^2 - 9x + 10 = 0$$
$$(2x - 5)(x - 2) = 0$$
$$x = 2.5 \text{ or } x = 2$$

11. a) $x = 0.80$
b) $x = -6.91$
c) $x = 3.16$
d) $x = 0.34$

12. $\log_5 [(x - 1)(x - 2)] = \log_5 (x + 6)$

$$(x - 1)(x - 2) = x + 6$$
$$x^2 - 3x + 2 = x + 6$$
$$x^2 - 4x - 4 = 0$$

Using the quadratic formula

$$x = \frac{4 \pm \sqrt{16 - (4)(-4)}}{2}$$
$$x = \frac{4 \pm \sqrt{32}}{2}$$

$x = 4.83$ or $x = -0.83$

As x must be >2, -0.83 is extraneous; $x = 4.83$

13. $\log_3 (-8) = x$; $3^x = -8$; Raising positive 3 to any power produces a positive value. If $x \geq 1$, then $3^x \geq 3$. If $0 \leq x < 1$, then $1 \leq x < 3$. If $x < 0$, then $0 < x < 1$.

14. a) $x > 3$

b) If x is 3, we are trying to take the logarithm of 0. If x is less than 3, we are trying to take the logarithm of a negative number.

15. $\frac{1}{2}(\log x + \log y) = \frac{1}{2} \log xy = \log \sqrt{xy}$ so $\frac{x + y}{5} = \sqrt{xy}$ and $x + y = 5\sqrt{xy}$. Squaring both sides gives $(x + y)^2 = 25xy$. Expanding gives $x^2 + 2xy + y^2 = 25xy$; therefore, $x^2 + y^2 = 23xy$.

16. $\log (35 - x^3) = 3 \log (5 - x)$

$$\log (35 - x^3) = \log (5 - x)^3$$
$$35 - x^3 = (5 - x)^3$$
$$35 - x^3 = -x^3 + 15x^2 - 75x + 125$$
$$15x^2 - 75x + 90 = 0$$
$$x^2 - 5x + 6 = 0$$
$$(x - 3)(x - 2) = 0$$
$$x = 3 \text{ or } x = 2$$

17. $\log_2 a + \log_2 b = 4$; $\log_2 ab = 4$; $2^4 = ab$; $16 = ab$. The values of a and b that satisfy the original equation are pairs that have a product of 16, but a and b must also both be positive. The possible pairs are: 1 and 16, 2 and 8, 4 and 4, 8 and 2, and 16 and 1.

18. $\log_2 (5x + 4) = 3 + \log_2 (x - 1)$

$$\log_2 (5x + 4) - \log_2 (x - 1) = 3$$
$$\log_2 \frac{5x + 4}{x - 1} = 3$$
$$2^3 = \frac{5x + 4}{x - 1}$$
$$8(x - 1) = 5x + 4$$
$$8x - 8 = 5x + 4$$
$$3x = 12$$
$$x = 4$$

Substituting 4 in for x in the first equation

$$y = \log_2((5)(4) + 4)$$
$$y = \log_2 24$$
$$2^y = 24$$
$$\log 2^y = \log 24$$
$$y \log 2 = \log 24$$
$$0.301y = 1.38$$
$$y = 4.58$$

19. a) $5^0 = \log_3 x$

$\log_3 x = 1$

$3^1 = x$

$x = 3$

b) $2^1 = \log_4 x$

$\log_4 x = 2$

$4^2 = x$

$x = 16$

20. $(2^{-1})^{x+y} = 2^4$ $\qquad$ $(x - y)^{-3} = 8$

$2^{-x-y} = 2^4$ $\qquad$ $(x - y)^{-3} = \left(\frac{1}{2}\right)^{-3}$

$-x - y = 4$ $\qquad$ $x - y = \frac{1}{2}$

Adding the two equations gives

$-2y = 4\frac{1}{2}$

$y = -2.25$

Substituting into the first equation

$-x + 2.25 = 4$

$-x = 1.75$

$x = -1.75$

8.7 Solving Problems with Exponential and Logarithmic Functions, pp. 499–501

1. First earthquake: $5.2 = \log x$; $10^{5.2} = 158\,489$
Second earthquake; $6 = \log x$; $10^6 = 1\,000\,000$
Second earthquake is 6.3 times stronger than the first.

2. $\text{pH} = -\log(\text{H}^+)$

$\text{pH} = -\log 6.21 \times 10^{-8}$

$\text{pH} = -(-7.2)$

$\text{pH} = 7.2$

3. $1\,000\,000 \times 10^{-12}\ \text{W/m}^2 = 10^{-6}\ \text{W/m}^2$; the intensity of the sound

$$L = 10 \log \frac{10^{-6}}{10^{-12}}$$
$$L = 10 \log 10^6$$
$$L = (10)(6) = 60\ \text{dB}$$

4. $69 = 10 \log \frac{I}{10^{-12}}$ $\qquad$ $60 = 10 \log \frac{I}{10^{-12}}$

$6.9 = \log I - \log 10^{-12}$ $\qquad$ $6 = \log I - \log 10^{-12}$

$6.9 = \log I + 12$ $\qquad$ $6 = \log I + 12$

$-5.1 = \log I$ $\qquad$ $-6 = \log I$

$10^{-5.1} = I$ $\qquad$ $10^{-6} = I$

$I = 7.9 \times 10^{-6}$ $\qquad$ $I = 1 \times 10^{-6}$

A heavy snore is 7.9 times louder than a normal conversation.

5. a) $9 = -\log \text{H}$

$-9 = \log \text{H}$

$10^{-9} = 0.000\,000\,001 = \text{H}$

b) $6.6 = -\log \text{H}$

$-6.6 = \log \text{H}$

$10^{-6.6} = 0.000\,000\,251 = \text{H}$

c) $7.8 = -\log \text{H}$

$-7.8 = \log \text{H}$

$10^{-7.8} = 0.000\,000\,016 = \text{H}$

d) $13 = -\log \text{H}$

$-13 = \log \text{H}$

$10^{-13} = 0.000\,000\,000\,000\,1 = \text{H}$

6. a) $\text{pH} = -\log 0.000\,32 = 3.49$

b) $\text{pH} = -\log 0.000\,3 = 3.52$

c) $\text{pH} = -\log 0.000\,045 = 4.35$

d) $\text{pH} = -\log 0.005 = 2.30$

7. a) $\text{pH} = -\log 10^{-7} = 7$

b) Tap water is more acidic than distilled water as it has a lower pH than distilled water (pH 7).

8. $109 = 10 \log \frac{I}{10^{-12}}$ $\qquad$ $118 = 10 \log \frac{I}{10^{-12}}$

$10.9 = \log I - \log 10^{-12}$ $\qquad$ $11.8 = \log I - \log 10^{-12}$

$10.9 = \log I + 12$ $\qquad$ $11.8 = \log I + 12$

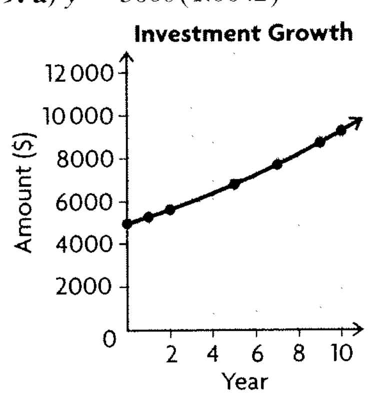
$-1.1 = \log I$ $\qquad$ $-0.2 = \log I$

$10^{-1.1} = I$ $\qquad$ $10^{-0.2} = I$

$I = 0.079$ $\qquad$ $I = 0.63$

An amplifier is 7.98 times louder than a lawn mower.

9. a) $y = 5000(1.0642)^t$

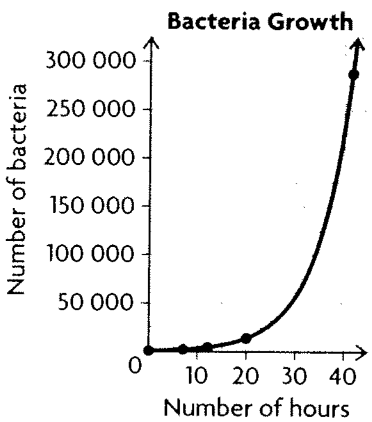


b) 6.42%

c)
$$10\,000 = 5000(1.0642)^t$$
$$2 = 1.0642^t$$
$$\log 2 = \log 1.0642^t$$
$$\log 2 = t \log 1.0642$$
$$\frac{\log 2}{\log 1.0642} = t$$
$$t = 11.14 \text{ years}$$

10.
$$4.2 = 10^{1-0.13x}$$
$$\log 4.2 = \log 10^{1-0.13x}$$
$$\log 4.2 = (1 - 0.13x)\log 10$$
$$0.623 = 1 - 0.13x$$
$$-0.376 = -0.13x$$
$$2.90 = x$$

11. a) Average growth is 15%.
Equation is $y = 850(1.15)^x$



b)
$$1700 = 850(1.15)^x$$
$$2 = 1.15^x$$
$$\log 2 = \log 1.15^x$$
$$\log 2 = x \log 1.15$$
$$0.301 = x(0.061)$$
$$x = 4.9 \text{ h}$$

12. a) 1.22, 1.43, 1.69, 2.00, 2.18, 2.35

b) Add the growth factors and divide by 7 = 1.81

c) $w = 5.061\,88(1.0618)^t$

d) $w = 5.061\,88(1.0618)^t$

e)
$$2 = (1.0618)^t$$
$$\log 2 = t \log 1.0618$$
$$0.301 = 0.026t$$
$$t = 11.5\ ^\circ\text{C}$$

13.
$$0.5 = (0.979)^x$$
$$x = \text{number of cycles}$$
$$\log 0.5 = \log 0.979^x$$
$$\log 0.5 = x \log 0.979$$
$$-0.301 = -0.009x$$
$$33 = x$$
33 cycles

14.
$$4000 = 2500(1.065)^t$$
$$1.6 = (1.065)^t$$
$$\log 1.6 = \log 1.065^t$$
$$\log 1.6 = t \log 1.065$$
$$0.204 = 0.0273t$$
$$7.4 = t$$
7.4 years

15.
$$20 = 80(10^{-0.023t})$$
$$0.25 = 10^{-0.023t}$$
$$\log 0.25 = \log 10^{-0.023t}$$
$$\log 0.25 = (-0.023t)\log 10$$
$$-0.602 = -0.023t$$
$$26.2 = t$$
26.2 days

16. Answers may vary. For example: (1) Tom invested \$2000 in an account that accrued interest, compounded annually, at a rate of 6%. How long will it take for Tom's investment to triple? (2) Indira invested \$5000 in a stock that made her \$75 every month. How long will it take her investment to triple?
The first problem could be modelled using an exponential function. Solving this problem would require the use of logarithms. The second problem could be modelled using a linear equation. Solving the second problem would not require the use of logarithms.

17.
$$70 = 10 \log \frac{I}{10^{-12}}$$
$$7 = \log I - \log 10^{-12}$$
$$7 = \log I + 12$$
$$-5 = \log I$$
$$10^{-5} = I$$
$$I = 0.000\,01$$
Therefore, the intensity of the sound of the second car is 0.000 02.
$$x = 10 \log \frac{0.000\,02}{10^{-12}}$$
$$x = \log 20\,000\,000$$
$$x = 73 \text{ dB}$$

18. a) $C = P(1.038)^t$ where P is the present cost of goods and services and t is the number of years.

b)
$$C = 400(1.038)^{10}$$
$$C = \$580.80$$

c)
$$47.95 = P(1.038)^{10}$$
$$47.95 = P(1.45)$$
$$P = \$33.07$$

8.8 Rates of Change in Exponential and Logarithmic Functions, pp. 507–508

1. a) $\frac{16 - 75}{10 - 2} = -7.375$

b) $\frac{32 - 125}{5 - 1} = -23.25$

c) $\frac{10 - 16}{13 - 10} = -2$

2. The instantaneous rate of decline was greatest in year 1. The negative change from year 1 to year 2 was 50, which is greater than the negative change in any other two-year period.

3. Use the equation $y = (96.313)(0.8296)^x$, which was obtained from doing an exponential regression on the data.

a) $y = (96.314)(0.8297)^{1.9}$ $y = (96.314)(0.8297)^{2.1}$
$y = 67.5360$ $y = 65.0593$

$$\frac{67.5520 - 65.0764}{1.9 - 2.1} = -12.378$$

b) $y = (96.314)(0.8297)^{6.9}$ $y = (96.314)(0.8297)^{7.1}$
$y = 26.5610$ $y = 25.5876$

$$\frac{26.5610 - 25.5876}{7.9 - 8.1} = -4.867$$

c) $y = (96.314)(0.8297)^{11.9}$
$y = 10.4436$
$y = (96.314)(0.8297)^{12.1}$
$y = 10.0608$

$$\frac{10.4436 - 10.0608}{11.9 - 12.1} = -1.914$$

4. a) $A(t) = 6000(1.075)^t$

b) $A(t) = 6000(1.075)^{9.9}$ $A(t) = 6000(1.075)^{10.1}$
$y = 12\,277.08$ $y = 12\,455.95$

$$\frac{12\,277.08 - 12\,455.95}{9.9 - 10.1} = 894.35$$

c) $A(t) = 6000(1.0375)^t$
$A(t) = 6000(1.0375)^{19.9}$ $A(t) = 6000(1.0375)^{20.1}$

$$\frac{12\,482.87 - 12\,575.12}{19.9 - 20.1} = 461.25$$

5. a) i) $y = 1000(1.06)^2$
$y = 1123.60$

$$\frac{1123.60 - 1000.00}{2 - 0} = 61.80$$

ii) $y = 1000(1.06)^5$
$y = 1338.23$

$$\frac{1338.23 - 1000.00}{5 - 0} = 67.65$$

iii) $y = 1000(1.06)^{10}$
$y = 1790.85$

$$\frac{1790.85 - 1000.00}{10 - 0} = 79.08$$

b) The rate of change is not constant because the value of the account each year is determined by adding a percent of the previous year's value.

6. a) $M(t) = 500(0.5)^{\frac{24}{5.2}} = 20.40$ g

b) $M(47.9) = 500(0.5)^{\frac{47.9}{5.2}}$ $M(48.1) = 500(0.5)^{\frac{48.1}{5.2}}$
$M(47.9) = 0.8434$ $M(48.1) = 0.8212$

$$\frac{0.8434 - 0.8212}{47.9 - 48.1} = -0.111 \text{ g/h}$$

7. a) $\frac{30.21 - 0.0002}{20 - 1} = 1.59$ g/day

b) $y = 0.0017(1.7698)^x$, where x is the number of days after the egg is laid

c) i) $y = 0.0017(1.7698)^{3.9}$ $y = 0.0017(1.7698)^{4.1}$
$y = 0.015\,75$ $y = 0.017\,65$

$$\frac{0.015\,75 - 0.017\,66}{3.9 - 4.1} = 0.0095 \text{ g/day}$$

ii) $y = 0.0017(1.7698)^{11.9}$ $y = 0.0017(1.7698)^{12.1}$
$y = 1.5162$ $y = 1.6995$

$$\frac{1.5162 - 1.6995}{11.9 - 12.1} = 0.917 \text{ g/day}$$

iii) $y = 0.0017(1.7698)^{19.9}$ $y = 0.0017(1.7698)^{20.1}$
$y = 145.93$ $y = 163.58$

$$\frac{145.93 - 163.58}{19.9 - 20.1} = 88.25 \text{ g/day}$$

d) $6 = 0.0017(1.7698)^x$
$3529 = 1.7698^x$
$\log 3529 = \log 1.7698^x$
$\log 3529 = x \log 1.7698$
$3.548 = 0.248x$
$x = 14.3$ days

8. a) $50 = 100(1.2)^{-t}$
$0.5 = (1.2)^{-t}$
$\log 0.5 = \log (1.2)^{-t}$
$\log 0.5 = -t \log (1.2)$
$-0.301 = -(0.079)t$
$t = 3.81$

b) $y = 100(1.2)^{-3.80}$ $y = 100(1.2)^{-3.82}$
$y = 50.02$ $y = 49.83$

$$\frac{50.02 - 49.83}{3.80 - 3.82} = 9.5\ \%/\text{year}$$

9. a) $y = 12\,000(0.982)^t$

b) $y = 12\,000(0.982)^{9.9}$ $\quad y = 12\,000(0.982)^{10.1}$
$y = 10\,025.01$ $\quad y = 9988.66$

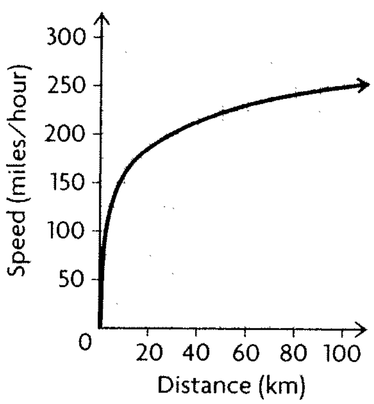
$$\frac{10\,025.01 - 9988.66}{9.9 - 10.1} = -181.7 \text{ people/year}$$

c) $6000 = 12\,000(0.982)^t$
$0.5 = (0.982)^t$
$\log 0.5 = \log (0.982)^t$
$\log 0.5 = t \log 0.982$
$-0.301 = -0.0079t$
$38.1 = t$ (yrs for pop to decrease by half)
$y = 12\,000(0.982)^{38.0}$ $\quad y = 12\,000(0.982)^{38.2}$
$y = 6017.5$ $\quad y = 5995.7$

$$\frac{6017.5 - 5995.7}{38.0 - 38.2} = -109 \text{ people/year}$$

10. Both functions approach a horizontal asymptote. Each change in x yields a smaller and smaller change in y. Therefore, the instantaneous rate of change grows increasingly small, toward 0, as x increases.

11. a)



b) $S(d) = 93 \log 10 + 65$; $S(d) = 158$ mph
$S(d) = 93 \log 100 + 65$; $S(d) = 251$ mph
251 mph − 158 mph = 93 mph
93 mph/90 m = 1.03 miles/hour/hour

c) $S(d) = 93 \log 9.9 + 65$ $\quad S(d) = 93 \log 10.1 + 65$
$S(9.9) = 157.59$ $\quad S(10.1) = 158.40$

$$\frac{157.59 - 158.40}{9.9 - 10.1} = 4.03 \text{ miles/hour/hour}$$

$S(d) = 93 \log 99.9 + 65$ $\quad S(d) = 93 \log 100.1 + 65$
$S(99.9) = 250.96$ $\quad S(100.1) = 251.04$

$$\frac{250.96 - 251.04}{99.9 - 100.1} = 0.403 \text{ miles/hour/hour}$$

d) The rate at which the wind changes during shorter distances is much greater than the rate at which the wind changes at farther distances. As the distance increases, the rate of change approaches 0.

12. To calculate the instantaneous rate of change for a given point, use the exponential function to calculate the values of y that approach the given value of x. Do this for values on either side of the given value of x. Determine the average rate of change for these values of x and y. When the average rate of change has stabilized to a constant value, this is the instantaneous rate of change.

13. a) and **b)** Only a and k affect the instantaneous rate of change. Increases in the absolute value of either parameter tend to increase the instantaneous rate of change.

Chapter Review, pp. 510–511

1. a) $x = 4^y$; $y = \log_4 x$
b) $x = a^y$; $y = \log_a x$
c) $x = \left(\frac{3}{4}\right)^y$; $y = \log_{\frac{3}{4}} x$
d) $q = P^m$; $m = \log_p q$

2. a) $a = -3$; $k = 2$; vertical stretch by a factor of 3, reflection in the x-axis, horizontal compression by a factor of $\frac{1}{2}$
b) $d = 5$; $c = 2$; horizontal translation 5 units to the right, vertical translation 2 units up
c) $a = \frac{1}{2}$; $k = 5$; vertical compression by a factor of $\frac{1}{2}$, horizontal compression by a factor of $\frac{1}{5}$
d) $k = -\frac{1}{3}$; $c = -3$; horizontal stretch by a factor of 3, reflection in the y-axis, vertical shift 3 units down

3. a) $a = \frac{2}{5}$; $c = -3$; $y = \frac{2}{5} \log x - 3$
b) $a = -1$; $k = \frac{1}{2}$; $d = 3$; $y = -\log \left[\frac{1}{2}(x - 3)\right]$
c) $a = 5$; $k = -2$; $y = 5 \log(-2x)$
d) $k = -1$; $d = 4$; $y = \log(-x - 4) - 2$

4. Compared to $y = \log x$, $y = 3 \log (x - 1) + 2$ is vertically stretched by a factor of 3, horizontally translated 1 unit to the right, and vertically translated 2 units up.

5. a) $7^x = 343$; $7^x = 7^3$; $x = 3$
b) $\left(\frac{1}{2}\right)^x = 25$; $\left(\frac{1}{2}\right)^x = \left(\frac{1}{2}\right)^{-2}$; $x = -2$
c) $19^x = 1$; $19^x = 19^0$; $x = 0$
d) $4^x = \frac{1}{256}$; $4^x = 4^{-4}$; $x = -4$

6. a) $3^x = 53$
$\log 3^x = \log 53$
$x \log 3 = \log 53$

$0.477x = 1.724$
$x = 3.615$

b) $4^x = \frac{1}{10}$
$\log 4^x = \log \frac{1}{10}$
$x \log 4 = \log \frac{1}{10}$
$0.602x = -1$
$x = -1.661$

c) $6^x = 159$
$\log 6^x = \log 159$
$x \log 6 = \log 159$
$0.778x = 2.201$
$x = 2.829$

d) $15^x = 1456$
$\log 15^x = \log 1456$
$x \log 15 = \log 1456$
$1.176x = 3.163$
$x = 2.690$

7. a) $\log [(5)(11)] = \log 55$

b) $\log \frac{20}{4} = \log 5$

c) $\log_5 \frac{(6)(8)}{12} = \log_5 4$

d) $\log 3^2 + \log 2^4 = \log 9 + \log 16$
$= \log [(9)(16)]$
$= \log 128$

8. a) $\log_6 \frac{42}{7} = \log_6 6$
$= 1$

b) $\log_3 \frac{(5)(18)}{10} = \log_3 9$
$= 2$

c) $\frac{1}{3} \log_7 49$; $\log_7 49$ in exponential form: $7^x = 49$, $x = 2$, so $\frac{1}{3} \log_7 49 = \left(\frac{1}{3}\right)(2) = \frac{2}{3}$

d) $\log_4 8^2 = \log_4 64$
$= 3$

9. It is shifted 4 units up.

10. a) $5^x = 5^5$; $x = 5$

b) $\log 4^x = \log 16 + \log \sqrt{128}$
$x \log 4 = \log 16 + \frac{1}{2} \log 128$
$0.602x = 1.204 + 1.054$
$0.602x = 2.258$
$x = 3.75$

c) $4^{5x} = (4^2)^{2x-1}$
$4^{5x} = 4^{4x-2}$
$5x = 4x - 2$
$x = -2$

d) $(3^{5x})(3^2)^2 = 3^3$
$3^{5x+4} = 3^3$
$5x + 4 = 3$
$x = -0.2$

11. a) $\log 6^x = \log 78$
$x \log 6 = \log 78$
$0.778x = 1.892$
$x = 2.432$

b) $\log (5.4)^x = \log 234$
$x \log (5.4) = \log 234$
$0.732x = 2.369$
$x = 3.237$

c) $\log (8)(3^x) = \log 132$
$\log 8 + \log 3^x = \log 132$
$\log 8 + x \log 3 = \log 132$
$0.903 + 0.477x = 2.121$
$0.477x = 1.218$
$x = 2.553$

d) $(1.23)^x = 2.7$
$\log (1.23)^x = \log 2.7$
$x \log 1.23 = \log 2.7$
$0.0899x = 0.4314$
$x = 4.799$

12. a) Multiple through by 4^x: $4^{2x} + 6 = 5(4^x)$
$4^{2x} - 5(4^x) + 6 = 0$
$(4^x - 3)(4^x - 2) = 0$

$4^x - 3 = 0$		$4^x - 2 = 0$
$4^x = 3$		$4^x = 2$
$x \log 4 = \log 3$		$x \log 4 = \log 2$
$0.602x = 0.477$		$0.602x = 0.301$
$x = 0.79$	or	$x = 0.5$

b) $8(5^{2x}) + 8(5^x) - 6 = 0$
$(4(5^x) - 2)(2(5^x) + 3) = 0$

$4(5^x) - 2 = 0$	$2(5^x) + 3 = 0$
$4(5^x) = 2$	$2(5^x) = -3$
$5^x = \frac{1}{2}$	$5^x = -\frac{3}{2}$
$\log 5^x = \log \frac{1}{2}$	(no solution)

$x \log 5 = \log \frac{1}{2}$
$0.699x = -0.301$
$x = -0.43$

13. $7 = 20(0.5)^t$
$0.35 = (0.5)^t$
$\log 0.35 = t \log 0.5$
$-0.456 = -0.301t$
$t = (1.52)(3.6) = 5.45$ days

14. a) $5^3 = 2x - 1$
$125 = 2x - 1$
$x = 63$

b) $10^4 = 3x$
$10\,000 = 3x$
$x = \frac{10\,000}{3}$

c) $\log_4(3x - 5) = \log_4[(11)(2)]$
$\log_4(3x - 5) = \log_4 22$
$3x - 5 = 22$
$3x = 27$
$x = 9$

d) $\log(4x - 1) = \log[2(x + 1)]$
$\log(4x - 1) = \log(2x + 2)$
$4x - 1 = 2x + 2$
$2x = 3$
$x = 1.5$

15. a) $\log\frac{(x + 9)}{x} = 1$
$10^1 = \frac{(x + 9)}{x}$
$10x = x + 9$
$9x = 9$
$x = 1$

b) $\log[x(x - 3)] = 1$
$\log(x^2 - 3x) = 1$
$10^1 = x^2 - 3x$
$x^2 - 3x - 10 = 0$
$(x - 5)(x + 2) = 0$
$x = 5$ or $x = -2$
As x must be > 3, $x = -2$ is inadmissible: $x = 5$

c) $\log(x - 1)(x + 2) = 1$
$\log(x^2 + x - 2) = 1$
$10^1 = x^2 + x - 2$
$x^2 + x - 12 = 0$
$(x + 4)(x - 3) = 0$
$x = -4$ or $x = 3$
As x must be > 1, $x = -4$ is inadmissible: $x = 3$

d) $\frac{1}{2}\log(x^2 - 1) = 2$
$\log(x^2 - 1) = 4$
$10^4 = x^2 - 1$
$10\,001 = x^2$
$x = \pm\sqrt{10\,001}$

16. $100 = 10\log\left(\frac{I}{10^{-12}}\right)$
$10 = \log\left(\frac{I}{10^{-12}}\right)$
$10 = \log I - \log 10^{-12}$
$10 = \log I + 12$
$\log I = -2$
$I = 10^{-2}\ \text{W/m}^2$

17. $82 = 10\log\left(\frac{I}{10^{-12}}\right)$
$8.2 = \log\left(\frac{I}{10^{-12}}\right)$
$8.2 = \log I - \log 10^{-12}$
$8.2 = \log I + 12$
$\log I = -3.8$
$I = 10^{-3.8}\ \text{W/m}^2$

18. $\frac{10^{6.2}}{10^{5.5}} = 5$ times

19. $\frac{10^7}{10^{6.4}} = 3.9$ times

20. $\frac{10^{4.7}}{10^{2.3}} = 251.2$ $\qquad \frac{10^{12.5}}{10^{10.1}} = 251.2$

The relative change in each case is the same. Each change produces a solution with concentration 251.2 times the original solution.

21. It is exponential as there is a common ratio ($\doteq 5$) when comparing the y-values.
Doing an exponential regression gives $y = 3(2.25^x)$

22. $15\,000 = 20\,000(0.984)^t$
$0.75 = 0.984^t$
$\log 0.75 = \log 0.984^t$
$\log 0.75 = t\log 0.984$
$-0.125 = -0.007t$
$t = 17.8$ years

23. a) $\frac{514\,013 - 132\,459}{1994 - 1950} = 8671$ people per year

b) $\frac{345\,890 - 132\,459}{1980 - 1950} = 7114$

The rate of growth for the first 30 years is slower than the rate of growth for the entire period.

c) Doing an exponential regression
$y = 134\,322(1.03^x)$, where x is the number of years after 1950

d) i) $y = 134\,322(1.03^{19.9})$ $\qquad y = 134\,322(1.03^{20.1})$
$y = 241\,884$ $\qquad y = 243\,318$
$\frac{241\,884 - 243\,318}{19.9 - 20.1} = 7171$ people per year

ii) $y = 134\,322(1.03^{39.9})$ $\qquad y = 134\,322(1.03^{40.1})$
$y = 436\,870$ $\qquad y = 439\,460$
$\frac{436\,870 - 439\,460}{39.9 - 40.1} = 12\,950$ people per year

24. a) The increases from year to year are not the same, so a linear model is not best. Doing an exponential regression gives $y = 23(1.17^x)$, where x is the number of years since 1998.

b) $y = 23(1.17^{17})$

331 808 DVD player owners

c) Answers may vary. For example, I assumed that the rate of growth would be the same through 2015. This is not reasonable. As more people buy the players, there will be fewer people remaining to buy them, or newer technology may replace them.

d) $\frac{43 - 27}{4 - 1} = 5.3$ about 5300 DVD players per year

e) $y = 23(1.17^{1.9})$ $\quad y = 23(1.17^{2.1})$

$y = 30.99$ $\quad y = 31.98$

$\frac{30.99 - 31.98}{1.9 - 2.1} = 4.95$ about 4950 DVD players per year

f) Answers may vary. For example, the prediction in part e) makes sense because the prediction is for a year covered by the data given. The prediction made in part b) does not make sense because the prediction is for a year that is beyond the data given, and conditions may change, making the model invalid.

Chapter Self-Test, p. 512

1. a) $x = 4^y$; $\log_4 x = y$

b) original function in exponential form: $x = 6^y$

inverse $y = 6^x$; $\log_6 y = x$

2. a) $k = 2$; $d = 4$; $c = 3$; horizontal compression by a factor of $\frac{1}{2}$, horizontal translation 4 units to the right, vertical translation 3 units up

b) $a = -\frac{1}{2}$; $d = -5$; $c = -1$; vertical compression by a factor of $\frac{1}{2}$, reflection in the x-axis, horizontal translation 5 units to the left, vertical translation 1 unit down

3. a) $3^x = \frac{1}{9}$; $3^x = 3^{-2}$; $x = -2$

b) $\log_5 \frac{100}{4}$

$\log_5 25$

$5^x = 25$

$x = 5$

4. a) $\log \frac{(15 \times 40)}{6}$

$\log 100 = 2$

b) $\log_7 343 + \log_7 49^2$

$\log_7 (343)(2401)$

$\log_7 823\,543$

$7^x = 823\,543$

$7^x = 7^7$

$x = 7$

5. $\log_4 x^2 + \log_4 \left[(y)\left(\frac{1}{3}\right)(3)\right] - \log_4 x$

$= \log_4 x^2 + \log_4 y - \log_4 x$

$= \log_4 \frac{x^2 y}{x}$

$= \log_4 xy$

6. $\log 5^{x+2} = \log 6^{x+1}$

$(x + 2)\log 5 = (x + 1)\log 6$

$0.699(x + 2) = 0.778(x + 1)$

$x + 2 = 1.113x + 1.113$

$0.887 = 0.113x$

$x = 7.85$

7. a) $\log_4 (x + 2)(x - 1) = 1$

$\log_4 (x^2 + x - 2) = 1$

$4^1 = x^2 + x - 2$

$4 = x^2 + x - 2$

$x^2 + x - 6 = 0$

$(x + 3)(x - 2) = 0$

$x = -3$ or $x = 2$

As x must be > 1, $x = -3$ is inadmissible; $x = 2$

b) $\log_3 [(8x - 2)(x - 1)] = 2$

$\log_3 (8x^2 - 10x + 2) = 2$

$3^2 = 8x^2 - 10x + 2$

$8x^2 - 10x - 7 = 0$

$x = \frac{10 \pm \sqrt{100 - (4)(8)(-7)}}{16}$

$x = \frac{28}{16} = 1\frac{3}{4}$ or $x = -\frac{1}{2}$

As x must be > 1, $x = -\frac{1}{2}$ is inadmissible; $x = 1\frac{3}{4}$

8. a) 50g

b) $A(t) = 100(0.5)^{\frac{t}{5730}}$

c) $80 = 100(0.5)^{\frac{t}{5730}}$

$0.8 = (0.5)^{\frac{t}{5730}}$

$\log 0.8 = \log (0.5)^{\frac{t}{5730}}$

$\log 0.8 = \frac{t}{5730} \log (0.5)$

$-0.0969 = \left(\frac{t}{5730}\right)(-0.301)$

$t = 1844$ years

d) $A(t) = 100(0.5)^{\frac{99.9}{5730}}$ $A(t) = 100(0.5)^{\frac{100.1}{5730}}$

$A(t) = 98.799$ $A(t) = 98.796$

$$\frac{98.799 - 98.796}{99.9 - 100.1} = -0.015 \text{ g/year}$$

9. a) $t = \log\left(\frac{35 - 22}{75}\right) \div \log(0.75)$

$$t = \frac{-0.761}{-0.125} = 6 \text{ min}$$

b) $0 = \log\left(\frac{T - 22}{75}\right) \div \log(0.75)$

$$0 = \log\left(\frac{T - 22}{75}\right)$$

$$10^0 = \frac{T - 22}{75}$$

$$75 = T - 22$$

$$T = 97°$$